

Computeralgebra (2013)-Aalborg Universitet

Spiseseddel 17

17. gang, tirsdag d. 12. november, 8:15-12:00 i lokale G5-110

- 8:15-10:00 Forelæsning: Buchberger algorithm, Geometric applications (pages 608–617).
- 10:00-12:00 Arbejde i grupper: Opgaver fra [GG]: A, B, C, D, E, F, G, H, I, J, 21.21, 21.23, 21.17, 21.9 (kun i).

Opgave A: Let $R = \mathbb{F}_3[X, Y]$. Let $f = X^2Y + 2XY^2 + XY + X$, $f_1 = X + 2Y^2 + 1$, $f_2 = Y^2 + Y$. Divide f by $\{f_1, f_2\}$ considering the monomial order $<_{\text{lex}}$. Divide f by $\{f_1, f_2\}$ considering now the monomial order $<_{\text{grlex}}$.

Opgave B: Investigate how to define monomial orders in Maple or Sage.

Opgave C: Compute, in Maple or Sage, the S-polynomial of two polynomials in $\mathbb{F}_q[x_1, x_2, x_3, x_4]$.

Opgave D: Compute, in Maple or Sage, a Gröbner basis of an ideal in $\mathbb{F}_q[x_1, x_2, x_3, x_4]$ with respect to the 3 monomial orders considered in the lecture.

Opgave E: Compute a Gröbner basis (with Maple or Sage) of the following ideal with respect to the lexicographical ordering with $z > y > x$:

$$I = \langle x^5 + y^3 + z^2 - 1, x^2 + y^2 + z - 1, x^6 + y^5 + z^3 - 1 \rangle \subset \mathbb{Q}[x, y, z]$$

Opgave F: Compute a Gröbner basis (with Maple or Sage) of

$$I = \langle x^5 + y^4 + z^3 - 1, x^3 + y^2 + z^2 - 1 \rangle \subset \mathbb{Q}[x, y, z]$$

using lex and grevlex with $x > y > z$. Is there any difference?

Opgave G: Now we change one exponent in exercise F: Compute a Gröbner basis (with Maple or Sage) of

$$I = \langle x^5 + y^4 + z^3 - 1, x^3 + y^3 + z^2 - 1 \rangle \subset \mathbb{Q}[x, y, z]$$

using lex and grevlex with $x > y > z$. Is there any difference?

Opgave H: Compute a Gröbner basis (with Maple or Sage) of

$$I = \langle x^4 - yz^2w, xy^2 - z^3, x^3z - y^3w \rangle \subset \mathbb{Q}[x, y, z, w]$$

using lex and grevlex with $x > y > z > w$. Is there any difference (hint: grevlex is not always better than lex). Actually one can prove that $z^{n^2+1} - y^{n^2}w$ is in the reduced Gröbner basis of the ideal

$$\langle x^{n+1} - yz^{n-1}w, xy^{n-1} - z^n, x^nz - y^nw \rangle$$

w.r.t. grevlex with $x > y > z > w$.

Opgave I: Let $I = \langle f_1 = x^2y - 1, f_2 = xy^2 - x \rangle$ and consider the lex order.

1. Show that $\{f_1, f_2\}$ is not a Gröbner basis for I .

2. Trace the Buchberger algorithm for computing a Gröbner basis for I . You can use Sage or Maple for computing the S-polynomials.

Opgave J: Consider the ideal in Example 21.21 and the Gröbner basis $\{f_1, \dots, f_5\}$ computed in pages 608 and 609. Compute a minimal Gröbner basis this ideal using lemma 21.36. Use Maple or Sage to compute the reduced Gröbner basis of this ideal.

Med venlig hilsen,

Diego