Abstrakt algebra med konkrete anvendelser 2
Aalborg Universitet (2013)
Spiseseddel 3

3. gang , torsdag d. 26. september, 17:00–20:00 i lokale G5-112

- 17:00-18:00 Forelæsning: Gröbnerbasis (Kapitel 5 fra [Lau] og kapitel 21 fra [GG]). Gröbnerbasis med Sage (http://www.sagemath.org/).

- 18:00-20:00 Opgaveregning: A, B, C. Fra [GG]: 21.6, 21.2, 21.8, 21.7, 21.21, 21.23, 21.17, 21.9 (kun i). Fra [Lau]: 5.10, 5.13, 5.14, 5.15, 5.18, 5.19, 5.20, 5.21, 5.22.

  Exercise A: Trace the division algorithm for y^2x divided by $\{yx - y, y^2 - x\} \subset \mathbb{Q}[x, y]$ with respect grevlex with $y > x$.

  Exercise B:

  1. Let $I = \langle \{xy - x, -y + x^2\} \rangle \subset \mathbb{Q}[x, y]$ and consider the lex order with $x < y$. Show that $\{xy - x, -y + x^2\}$ is not a Gröbner basis with respect to the previous order.

  2. Compute a Gröbner basis of I with respect to the previous order using a command in Maple, Sage or Singular.

  3. Trace the Buchberger algorithm for computing a Gröbner basis for I with respect to the previous order.

  4. Compute a minimal Gröbner basis of I (using the Lemma 21.36 in [GG]).

  5. Compute the reduced Gröbner basis of I with respect to the previous order.

  Exercise C: Solve the following system of equations over $\mathbb{C}$ (computing a Gröbner base, not by using a command in Maple/Sage):

  $$\begin{cases} x^2 + y + z = 1 \\ x + y^2 + z = 1 \\ x + y + z^2 = 1 \end{cases}$$

  (Hint: there are 5 solutions).

Med venlig hilsen,

Diego