

Algebra 2 (2013)-Aalborg University

Lecture 20, April 18th

20th Lecture (A): Thursday April 18th, 8:15-12:00 at room G5-112.

- 8:15-8:45 Repetition from last lectures (pages 163–167 + 170–171).
- 8:45-10:45 Work in groups: Exercises from [Lau], 4.10 (page 179): A, B, C, D, 4.7, 4.27, 4.28, 4.29, 4.30.

Exercise A: Let $f \in F[X]$ be a reducible polynomial. Show that $F[X]/\langle f \rangle$ is not a field (hint: one can easily show that it is not a domain). For instance, find a zero divisor in $\mathbb{F}_2[X]/\langle X^2 + 1 \rangle$.

Exercise B: Find an irreducible polynomial $f \in \mathbb{F}_2[X]$ with degree 4. Construct $\mathbb{F}_{16} = \mathbb{F}_2[X]/\langle f \rangle$. Write the addition and multiplication table of \mathbb{F}_{16} (you do not have to write the complete table). Find a generator of the cyclic group \mathbb{F}_{16}^* .

Exercise C: Find an irreducible polynomial $f \in \mathbb{F}_4[X]$ with degree 2. Construct $\mathbb{F}_{16} = \mathbb{F}_4[X]/\langle f \rangle$. Write the addition and multiplication table of \mathbb{F}_{16} (you do not have to write the complete table). Find a generator of the cyclic group \mathbb{F}_{16}^* .

Exercise D: Prove that the fields constructed in exercises B and C are isomorphic.

- 10:45-12:00 Lecture: Existence and uniqueness of finite fields, (pages 171–173)

Best regards,

Diego