

# Algebra 2 (2013)-Aalborg University

## Lecture 19th, April 16th

**19th Lecture (C):** Tuesday April 16th, 8:15–12:00. I will not be present during this lecture, it is self-study.

During this lecture we will work on finite fields. The following exercises will help us to understand them. Work in groups, exercises A, B, C, D, E.

Exercise A: Find an irreducible polynomial  $f \in \mathbb{F}_2[X]$  with degree 2. Construct  $\mathbb{F}_4 = \mathbb{F}_2[X]/\langle f \rangle$ . Write the addition and multiplication table of  $\mathbb{F}_4$ . Find a generator of the cyclic group  $\mathbb{F}_4^*$ .

Exercise B: Find an irreducible polynomial  $f \in \mathbb{F}_2[X]$  with degree 4. Construct  $\mathbb{F}_{16} = \mathbb{F}_2[X]/\langle f \rangle$ . Write the addition and multiplication table of  $\mathbb{F}_{16}$ . Find a generator of the cyclic group  $\mathbb{F}_{16}^*$ .

Exercise C: Find an irreducible polynomial  $f \in \mathbb{F}_4[X]$  with degree 2. Construct  $\mathbb{F}_{16} = \mathbb{F}_4[X]/\langle f \rangle$ . Write the addition and multiplication table of  $\mathbb{F}_{16}$ . Find a generator of the cyclic group  $\mathbb{F}_{16}^*$ .

Exercise D: Prove that the fields constructed in exercises B and C are isomorphic.

Exercise E: Prove Lemma 4.8.1.

Best regards,

Diego