

# Algebra 2 (2012)-Aalborg University

## Lecture 7, February 23rd

**7th Lecture (B):** Thursday February 23rd, 8:15-12:00 at room G5-112.

- 8:15-10:00 Repetition from last lecture (pages 126–130). Lecture: Euclidean domains. Fermat's two-square theorem (130–134).
- 10:00-12:00 Work in groups: proofs + exercises.

Proofs: Proposition 3.3.2, Proposition 3.3.7, Theorem 3.3.9, Proposition 3.4.1, Corollary 3.4.2, Proposition 3.5.2, Proposition 3.5.3, Lemma 3.5.5, Proposition 3.5.6, Theorem 3.5.7, Proposition 3.5.9, Proposition 3.5.11, Theorem 3.5.15

Exercises from [Lau], 3.6 (page 138): 28, A, 29, 24, B, 23, C, D, 16, 13, 15, 8 (i and ii), 9, 10, 26, 24.

Exercise A: Prove that  $\mathbb{Z}[i]$  is a Euclidean domain (see pages 132 and 133).

Exercise B: Let  $R$  be an integral domain. Prove that for  $a, b \in R$ :  $ab \in R^*$  if and only if  $a, b \in R^*$ .

Exercise C: Check that the relation defined in page 123 is an equivalent relation and the two operations are well defined.

Exercise D: Let  $R$  be a ring. Prove that  $R$  is a field if and only if  $\langle 0 \rangle$  is a maximal ideal.

Best regards,

Diego