Afleveringsopgaver 4
Algebra 1, EVU 2011-Aalborg Universitet

These are the exercises that you can hand in, latest 2nd May in the morning. You need not solve them in the same order as they appear.

- Ex. A: Let H be a subgroup of G. Prove that if H has index 2 in G, then H is normal. Hint: consider the map $\Phi : G/H \to H\backslash G$ given by $\Phi(gH) = Hg^{-1}$, prove that it is well defined and it is bijective.
- Exercises 2.9.2, 2.9.3, 2.9.4 (page 132 in [KN]).
- Ex. B: We know that every subgroup of an abelian group is normal (Theorem 2, section 2.8). What about the converse? If every subgroup of a group G is normal in G, is G an abelian group?. The answer is negative. Prove that the quaternions Q (see page 123 in [KN]) are a nonabelian group and that every subgroup of Q is normal.
- Ex. C: Let $G = S_3$ and $H = \{e, (1\ 2\ 3), (1\ 3\ 2)\}$.
  1. Prove that H is a subgroup in G
  2. What is the index of H?
  3. Compute G/H and $G\backslash H$. Is H normal?
  4. Compute the Cayley table for G/H
  5. Prove that G/H is isomorphic to $\mathbb{Z}_2$
- Exercises 2.10.1, 2.10.3, 2.10.22, 2.10.31 (page 137 in [KN]).
- Ex. D: Prove (with detail) Theorem 5 from section 2.10.
- Ex. E: Prove (with detail) Theorem 1 and Theorem 2 of section 2.6.
- There are many examples in the book. However, most of them are given without detailed computations. You can consider these examples as exercises and then check that you get the same result, see for instance example 1 in section 2.9. You are welcome to hand in these examples as well.
- Exercises and examples from previous lectures.

Med venlig hilsen,

Diego