Algebra 1 (2012)-Aalborg University
Lecture 21, November 15th

21st Lecture: Thursday November 15th. I will not be present during this lecture.

8:15-12:00 Work in groups.

- Exercise A: Let $G = \langle g \rangle$, with $|g| = 12$ (Hint: note that G is cyclic and $|G| = 12$). Let $H = \langle g^4 \rangle$.
  1. Prove that H is a normal subgroup of G (Hint: use proposition 2.7.4 and/or exercise 2.14 in [Lau])
  2. Prove that $|H| = 3$. Compute $|G/H|$ (Hint: use Lagrange's Theorem).
  3. Compute the left-cosets in G/H.
  4. Compute the composition table of G/H.
  5. Is G/H cyclic?
  6. What is the order of a^2H in G/H?, What is the order of a^3H in G/H?
- Exercise B: Let $G = D_3 = \{e, a, a^2, b, ba, ba^2\}$, where $\text{ord}(a) = 3$, $\text{ord}(b) = 2$ and $aba = b$. (Hints: $a^k ba^k = b$, $a^k b = ba^{-k} = ba^{3-k}$ and $|ba^k| = 2$ for all $k \in \mathbb{Z}$)
  1. Compute the composition table of D_3.
  2. Let $H = \langle a \rangle$. Prove that H is a normal subgroup in G.
  3. Compute G/H.
  4. Compute the composition table of G/H.
  5. What is the order of every element of G/H.
  6. Prove that D_3 is isomorphic to S_3
  7. Compute $Z(G)$.
- Exercise C: Let G be a group, the center of G is

$$Z(G) = \{z \in G : zg = gz, \text{ for all } g \in G\}.$$

  1. Prove that $Z(G)$ is an abelian subgroup of G.
  2. Prove that $Z(G) = G$ is and only if G is abelian.
  3. Prove that $Z(G)$ is normal in G (actually, every subgroup of $Z(G)$ is normal in G).

Each group can write their solution for two exercises leave it in my mailbox (just one set of exercises per group). You are welcome to provide feedback about successes and difficulties.

Best regards,

Diego