Algebra 1 (2012)-Aalborg University
Lecture 18, November 6th

18th Lecture: Tuesday November 6th. I will not be present during this lecture.

- Lecture: This part will consist of self-study. The topic is Corollary 2.7.6 and section 2.8-Groups and numbers (pages 76–78). There are some slides for this lecture.

- Work in groups. Exercises: 32, 35, A, B, C, 31, D, E, 29, 34, 16, 17, 30.

  Exercise A: Let m and n be relatively prime positive integers. If G is a cyclic group of order mn, show that G is isomorphic to $H \times K$, where H and K are cyclic groups of orders m and n, respectively.

  Exercise B (from last year's exam):

  1. How many elements are there of order 11 in $\mathbb{Z}/32\mathbb{Z}$?
  2. Write down all the elements of order 8 in $\mathbb{Z}/32\mathbb{Z}$.
  3. How many subgroups are there of order 8 in $\mathbb{Z}/32\mathbb{Z}$?. Write down the subgroups of order 8 in $\mathbb{Z}/32\mathbb{Z}$.
  4. How many elements of $(\mathbb{Z}/11\mathbb{Z})^*$ are generators of $(\mathbb{Z}/11\mathbb{Z})^*$? (hint: you do not need to compute them).

  Exercise C (from last year's exam): Let $s = \mathrm{lcm}(m, n)$. Show that $\mathbb{Z}/s\mathbb{Z}$ is isomorphic to a subgroup of $\mathbb{Z}/m\mathbb{Z} \times \mathbb{Z}/n\mathbb{Z}$.

  Exercise D: Let G be the group $\{1, -1, i, -i\} \subset \mathbb{C}$. Prove that G is cyclic. Which are the generators of G?

  Exercise E: Let G be a cyclic group of order 12. Draw the lattice diagram with al the subgroups of G (a line can be drawn up from K to H whenever $K \subset H$).

  Each group can write their solution for two exercises and leave it in my mailbox (just one set of exercises per group). You can also welcome to provide feedback about successes and difficulties.

Best regards,

Diego