# Algebra 1 (2012)-Aalborg University
# Lecture 11, October 11th

**11th Lecture:** Thursday October 11th, 8:15-12:00 at room G5-112.

- 12:30-13:00 Repetition from last lecture. Normal subgroups. Quotient groups of the integers. The multiplicative group of prime residue classes (pages 64–67).
- 13:00-15:00 Work in groups. Exercises from [Lau], 2.11 (page 104): 19, 14, 13, 12 (only second question, hint: the answer is yes, consider $(\mathbb{Z}/8\mathbb{Z})^*$), A, B, 15, C, D, E, F.

  Exercise A: Let $G = S_3$. Prove that $N = \{e, d, f\}$ is a normal subgroup of $G$. What is the index of $N$?. Compute the composition table of $G/N$. Compare it with the composition table of $\mathbb{Z}/2\mathbb{Z}$

  Exercise B: Compare the groups $(\mathbb{Z}/8\mathbb{Z})^*$ and $\mathbb{Z}/4\mathbb{Z}$ with the groups in exercise 2.11.2. Are they "equal"?

  Exercise C: Let $G = \mathbb{Z}/6\mathbb{Z}$ and $H = \{[0], [3]\}$, compute $G/H$.

  Exercise D: Let $H$ be a finite nonempty subset of a group $G$. Prove that $H$ is a subgroup of $G$ if and only if $H$ is closed.

  Exercise E: Let $H$ and $K$ be subgroups of a group $G$. Show that $H \cap K$ is a subgroup of $G$.

  Exercise F: Let $G = D_3 = \{e, a, a^2, b, ba, ba^2\}$, where $\operatorname{ord}(a) = 3$, $\operatorname{ord}(b) = 2$ and $aba = b$. Let $H = \langle a \rangle$. Prove that $H$ is a normal subgroup in $G$. Write the composition table for $G/H$.

- 15:00-16:15 Lecture: Group homomorphisms. The isomorphism theorem (pages 68 + 70–72).

Best regards,

Diego