Algebra 1 (2011)-Aalborg University
Lecture 18, November 17th

18th Lecture: Thursday November 17th, 12:30-16:15 at room G5-112.
I will not be present during this lecture. The group G3-119 will be responsible for the lecture.

- 12:30-12:45 Short repetition. Group G3-119 will give this lecture. Normal subgroups and quotient group (pages 64–65).
- 12:45-15:30 Work in groups.
  - Exercise A: Let $G = \langle g \rangle$, with $|g| = 12$ (Hint: note that G is cyclic and $|G| = 12$). Let $H = \langle g^4 \rangle$.
    1. Prove that H is a normal subgroup of G (Hint: use proposition 2.7.4 and/or exercise 2.14 in [Lau])
    2. Prove that $|K| = 3$. Compute $|G/H|$ (Hint: use Lagrange's Theorem).
    3. Compute the left-cosets in G/H.
    4. Compute the composition table of G/H.
    5. Is G/H cyclic?
    6. What is the order of a^2H in G/H?, What is the order of a^3H in G/H?
  - Exercise B: Let G be a group, the center of G is

$$Z(G) = \{z \in G : zg = gz, \text{ for all } g \in G\}.$$

    1. Prove that $Z(G)$ is an abelian subgroup of G.
    2. Prove that $Z(G) = G$ is and only if G is abelian.
    3. Prove that $Z(G)$ is normal in G (actually, every subgroup of $Z(G)$ is normal in G).
  - Exercise C: Let $G = D_4 = \{1, a, a^2, a^3, b, ba, ba^2, ba^3\}$, where $|a| = 4$, $|b| = 2$ and $aba = b$.
    1. Compute the composition table of D_4 (Hints: $a^k b a^k = b$, $a^k b = ba^{-k} = ba^{4-k}$ and $|ba^k| = 2$ for all $k \in \mathbb{Z}$).
    2. Prove that $H = Z(D_4) = \{1, a^2\}$. Hence, H is normal in G (by previous exercise).
    3. Compute G/H.
    4. Compute the composition table of G/H.
    5. What is the order of every element of G/H.
    6. Prove that G/H is isomorphic to $(\mathbb{Z}/8\mathbb{Z})^*$

  Each group can write their solution for exercises A and C and leave it in my mailbox (just one set of exercises per group).

- 15:30-16:15 Common discussion in the lecture room directed by group G3-119. Discuss the solution of the exercises, especially exercises A and C. At the end of the lecture, group G3-119 will orientate the teacher, by e-mail, about the concrete successes and difficulties during the lecture.

Best regards,

Diego