Sums of Reciprocals of Polynomials Over Finite Fields

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Abstract

Consider the following example (typical in college algebra):

$$\frac{1}{x^2}+\frac{1}{x^2+1}+\frac{1}{x^2+x}+\frac{1}{x^2+x+1}=\frac{4x^5+6x^4+8x^3+6x^2+3x+1}{x^2(x+1)\,(x^2+1)\,(x^2+x+1)}.$$

Now let's assume that all the coefficients in the above are from the binary field $\mathbb{F}_2=\mathbb{Z}_2=\{0,1\}$. The result becomes much cleaner:

$$\frac{1}{x^2}+\frac{1}{x^2+1}+\frac{1}{x^2+x}+\frac{1}{x^2+x+1}=\frac{1}{(x^2+x)(x^4+x)}.$$

After a brief introduction to finite fields, we consider the sum of the reciprocals of all monic polynomials of a given degree over a finite field $\mathbb{F}_q$ each raised to the power of k. When $k\le q$, the sum has a surprisingly simple result due to mysterious cancellations that occur in the sum. We discuss this interesting phenomenon and its connection to a deeper problem.

The talk is based on a recent paper in the MAA Monthly: K. Hicks, X. Hou, G. L. Mullen, *Sums of reciprocals of polynomials over finite fields*, Amer. Math. Monthly, **119** (2012), 313 – 317.