The degree compatible Gröbner Fan for Linear Codes

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Abstract

The Gröbner fan of an ideal in the commutative polynomial ring consists of polyhedral cones indexing the different leading ideals and is thus the geometric collection of all reduced Gröbner bases for this ideal. One application of the Gröbner fan is the so-called Gröbner walk which is the conversion of Gröbner bases.

With the software system TiGERS (Toric Gröbner bases Enumeration by Reverse Search) an efficient alternative for computing the Gröbner fan has been provided for the special case of toric ideals. Indeed, by identifying a reverse search tree on the cones of the Gröbner fan, a memoryless combinatorial Gröbner walk can be established that furthermore, requires no cost weight vectors.

Linear codes, on the other hand, can be linked to this whole subject by associating to each linear code a binomial ideal that encodes the information about the code in the exponents. This correspondence proved to be very beneficial as it provided new approaches to several well-known problems in coding theory. Almost all applications, however, require the computation of a degree compatible Gröbner basis.

In this talk, it will be shown how methods from the software system TiGERS can be modified in order to compute all reduced Gröbner bases with respect to a degree compatible ordering for code ideals – even though these binomial ideals are not toric. To this end, the correspondence of linear codes and binomial ideals will be briefly described as well as their resemblance to toric ideals. Finally, we will hint at applications of the degree compatible Gröbner fan to the code equivalence problem.

Keywords
Linear code, Gröbner basis, Gröbner fan, code equivalence problem