On the Efficiency of Shortened Cyclic Multiple-Burst-Correcting Codes

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Abstract

On channels with memory, the noise is not independent from transmission to transmission. As a consequence, transmission errors occur in clusters or bursts, and channels with memory are called burst-error channels. Examples of burst-error channels are mobile telephony channels, where the error bursts are caused by signal fading owing to multipath transmission; cable transmission, which is affected by impulsive switching noise and crosstalk; and magnetic recording, which is subject to dropouts caused by surface defects and dust particles. Codes designed to correct burst errors are called burst-error correcting codes [8][9].

Hence, burst-correcting codes are of interest for some applications in which errors tend to occur in clusters. With higher transmission rates or higher storage densities this may even be more so in the future. The problem of correcting bursts of errors is a difficult one. In practice, Reed-Solomon codes, either interleaved or not, are used for correcting multiple bursts. However, it is of interest to find efficient multiple burst-correcting codes that are optimal in terms of redundancy.

In addition to the familiar Hamming distance, it is well known that there is also a burst distance and a burst weight [11]. An intuitive way to visualize the burst weight b of a vector $\underline{v}$, is by finding the minimum number of bursts of weight b that cover (cyclically) the nonzero coordinates of vector $\underline{v}$. We denote the burst weight b of $\underline{v}$ as $w_b(\underline{v})$. For instance, $w_2(100101101)=3$ and $w_4(100101101)=2$. The codes we consider are binary, thus, the burst distance b between vectors $\underline{u}$ and $\underline{v}$ is given by $d_b(\underline{u},\underline{v})=w_b(\underline{u}\oplus\underline{v})$.

The minimum burst-distance b of a code is denoted d_b. Notice that w_1 and d_1 represent the familiar Hamming weight and distance respectively. The codes considered in this talk are all binary and linear. Codes meeting the Singleton bound with equality are called Maximum Distance Separable (MDS). Other bounds can also be obtained using the burst distance [1][2], like for instance, the Hamming bound. Another well known bound for burst-correcting codes is the Gallager bound [3], which applies to both block and convolutional codes. In order to state the Gallager bound we need to recall the concept of guard space, which is essential in the study of bursts [3][8][9]. We used it extensively in [4] and will repeat this concept to make this talk self-contained.

Definition Assume that an all-zero sequence is transmitted and let $e_0, e_1, e_2 \ldots$ be the difference between the transmitted and the received sequences, i.e., 1s represent errors and 0s absence of errors. Then, a vector of consecutive b bits $(e_l, e_{l+1}, \ldots, e_{l+b-1})$ is called a burst of length b with respect to a guard space of length g if:

1. $e_l = e_{l+b-1} = 1$.
2. $b \leq g$.
3. The g bits preceding e_l and the g bits following e_{l+b-1} are all 0s (if $l < g$ then all the bits preceding l are 0). □

The Gallager bound states,

$$\frac{g}{b} \geq \frac{1+R}{1-R} \tag{1}$$

where R is the rate of the code ($R = k/n$ for block codes).

The Gallager bound is more general than the Reiger bound, since it applies to both block and convolutional codes, while the Reiger bound applies only to block codes. Even if we restrict only to block codes, the Gallager bound seems to be more general than the Reiger bound, since it connects the burst length with the guard space. However, the Reiger bound contains implicitly the guard space, although this does not look very clear from the bound itself. In fact, for block codes, both bounds are equivalent.

Shortened cyclic codes that are capable of correcting multiple bursts of errors are considered, together with tables of generator polynomials. Shortened cyclic codes that can correct all-around bursts as well as codes that cannot do so are presented, and under which conditions it is advantageous to use one or the other is studied. A bound that unifies both the Singleton and the Reiger bounds [10] is provided. A search algorithm based on Gray codes that extends a previous algorithm for searching one-burst correcting codes [5][6][7] is given.

Keywords

Burst errors, burst-correcting codes, cyclic codes, efficiency, error-correcting codes, Gallager bound, Gray codes, guard space, multiple burst-correcting codes, Reiger bound, Singleton bound, shortened cyclic codes.

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References

[1] G. Benelli, C. Bianciardi and V. Capellini, “Redundancy Bounds for Multiple-Burst Error-Correcting Codes,” Electronics Letters, Vol. 13, No. 13, pp 389-390, June 1977.

[2] G. Benelli, C. Bianciardi and V. Capellini, “Redundancy Bounds for Multiple Burst Error-Correcting Codes,” Alta Frequenza, Vol. XLVII, No. 9, September 1978.

[3] R. G. Gallager, “Information Theory and Reliable Communication,” McGraw-Hill, 1968.

[4] L. J. García Villalba, J. R. Fuentes Cortez and M. Blaum,“On the Efficiency of Shortened Cyclic Single-Burst-Correcting Codes,” IEEE Transactions on Information Theory, Vol. 56, No. 7 pp. 3290-96, July 2010.

[5] L. J. García Villalba, J. R. Fuentes Cortez, A. L. Sandoval Orozco and M. Blaum, “Use of Gray codes for Optimizing the Search of (Shortened) Cyclic Single Burst-Correcting Codes,” IEEE International Symposium on Information Theory, Saint-Petersburg, Russia, pp. 1186–1189, August 2011.

[6] L. J. García Villalba, J. R. Fuentes Cortez, A. L. Sandoval Orozco and M. Blaum, “Efficient Shortened Cyclic Codes Correcting Either Random Errors or Bursts,” IEEE Communications Letters, Vol. 15, No. 7, pp. 749-751, July 2011.

[7] L. J. García Villalba, J. R. Fuentes Cortez, A. L. Sandoval Orozco and M. Blaum, “An Efficient Algorithm for Searching Optimal Shortened Cyclic Single-Burst-Correcting Codes,” IEEE Communications Letters, Vol. 16, No. 1, pp. 89-91, January 2012.

[8] S. Lin and D. J. Costello, “Error Control Coding: Fundamentals and Applications,” Prentice Hall, 1983.

[9] S. Lin and D. J. Costello, “Error Control Coding (2nd Edition),” Prentice Hall, 2004.

[10] S. H. Reiger, “Codes for the Correction of ’Clustered’ Errors,” IRE Transactions on Information Theory, Vol. 6, No. 1, pp. 16–21, March 1960.

[11] S. Wainberg and J. K. Wolf, “Burst Decoding of Binary Block Codes on Q-ary Output Channels,” IEEE Transactions on Information Theory, Vol. 18, No. 5, pp. 684-686, September 1972.