Some results on finite fields

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Abstract

In the study of perfect sequences, use of trace functions in finite fields has been a common technique, dating back to Singer’s classical examples of m- sequences. Many variations of Singer’s theme have been the topic of research by many mathematicians during the last few decades. In all these investigations, expressing the underlying function as a polynomial whose coefficients are from the prime subfield has been a main problem. Explicit answers require the use of Stickelberger’s congruence of Gauss sums. Using elementary methods, we provide a simple result along these lines.

The following is well-known:

Let $f : F_{p^d}^{¿} \to F_{p^d}$ *be any function. Then* $f(x) = \sum_{k=0}^{p^d-2} \alpha_k x^k \in F_{p^d}[x]$

We prove a finer version of the above result:

Let p *be an odd prime. Let* $f(x) \in F_{p^d}[x]$ *. Let* $deg\, f(x) = r.$

Suppose $r < p^d - 1 \wedge f(\alpha) \in F_p$ *for each* $\alpha \in F_p^{¿}$ *. Then* $f(x) \in F_p[x].$