On a class of difference set pairs

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Let (G, +) be an abelian group of order v and let A and B be two subsets of G with $|A|=k \text{ and } |B|=k'$. If the list/multiset of differences $(x-y \pmod v) \vee x \in A, y \in B)$ contains every nonzero element of G exactly λ times, then (A,B) is a (v,k,k',e,λ) difference set pair (DSP) in G where $e=|A \cap B|$.

The case A = B reduces to the usual (v,k,λ) difference set.

In order for a difference set pair to exist, the parameters must clearly satisfy the relationship

$$k k' = \lambda(v-1)+e.$$

This is a necessary, but not a sufficient condition.

For odd v, a difference set pair is said to be balanced if $k=\frac{(v+1)}{2}$.

A difference set pair is said to be ideal if $v-2(k+k')+4\lambda=-1.$

We provide a new construction technique for DSPs using group rings. One such theorem is given below:

Theorem

Let G be an abelian group of order v. Let (A,B) be a balanced and ideal difference set pair in G with $A \subseteq B$ and parameters $\left(v, \frac{v+1}{2}, 2\lambda, 2\lambda, \lambda\right)$. Let H be an abelian group of prime-power order

$4m-1.$ Let *E* be a difference set in *H* with parameters $(4m-1, 2m, m)$. Define $C=EB$ and $D=EA+(H-E)(G-B)$. Then *(C,D)* is a balanced and ideal difference set pair in *G x H* with parameters $\left(v(4m-1), \frac{v(4m-1)+1}{2}, 4m\lambda, 4m\lambda, 2m\lambda\right)$.

Further new results and their connections to results of Peng, Xu, Arasu (2012) and Ke, Yu, Chang (2013) would be given. DSPs and their associated binary sequence pairs have applications in digital signal processing and cryptography.