On Generalized Lee Weight Codes over Dihedral Groups

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Abstract

In this contribution we show the structure on some codes over non-Abelian groups, namely over D_{2^m} the dihedral group of 2^m elements. We use the polycyclic presentation of D_{2^m} to give a natural extension of Lee metric in this case and propose a structure theorem for such codes.

Keywords

Codes over Groups, Polycyclic Codes, Dihedral Groups

Group codes are a generalization of linear codes which its underlying structure is defined over an alphabet given by a group. These codes were first studied by Slepian in [7]. It has been shown in [3] that Abelian group codes for the Hamming metric do not achieve the capacity of arbitrary channels. It has also been conjectured that non-Abelian group codes are inferior to Abelian group codes [1, 4, 3] in that case. Recently in [6] they proved that thre exist asymptotically good codes over non-abelian groups.

Whereas properties of group codes for Hamming metric have been extensively studied not to much is known in the non-abelian case for the Lee metric. Note that the Lee metric in the cyclic-group case has provide some nice and optimal non-linear binary codes as their Gray maps (see for example the seminal papers on this topic for block codes over $\mathbb{Z}_4$ the cyclic group with 4 elements [5, 2]).

The first step when dealing with non-abelian groups is consider the class of polycyclic groups. In this work we will consider codes over dihedral groups of 2^{m+1} elements. Based on the polycyclic representation $\texttt{pcp}(D_{2^m})$ of D_{2^m} we shall define the natural Lee metric on such codes that generalizes the well known Lee metric in the cyclic case. Based on the structure of $\texttt{pcp}(D_{2^m})$ we shall derive a canonical form of this type of codes based on a chosen set of generators.

References

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