Optimal optical orthogonal codes of weight 5 and small lengths

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Since the introduction of the fundamental principles of optical code-division multiple-access (OCDMA) using on-off pulses as signature sequences, the search for powerful code structures began. Among the most famous codes introduced to date are optical orthogonal codes (OOCs). They also have applications in mobile radio, frequency-hopping spread-spectrum communications, radar, sonar signal design, constructing protocol - sequence sets for the M-active-out-of T users collision channel without feedback, etc.

OOCs may also be viewed as constant weight error-correcting codes in which any two codewords are cyclically distinct. Also, some balanced incomplete block designs satisfy the requirements of an OOC.

For the basic concepts and notations concerning optical orthogonal codes and related designs we follow [1]. Let us denote by Z_v the ring of integers modulo v.

Definition 1 *A $(v, k, \lambda_a, \lambda_c)$ optical orthogonal code (OOC) can be defined as a collection $\mathcal{C} = \{C_1, \ldots, C_s\}$ of k-subsets (codeword-sets) of Z_v such that any two distinct translates of a codeword-set share at most λ_a elements while any two translates of two distinct codeword-sets share at most λ_c elements:*

$$|C_i \cap (C_i + t)| \leq \lambda_a, \quad 1 \leq i \leq s, \quad 1 \leq t \leq v - 1 \tag{1}$$

$$|C_i \cap (C_j + t)| \leq \lambda_c, \quad 1 \leq i < j \leq s, \quad 0 \leq t \leq v - 1. \tag{2}$$

The integers v and k are the length and the weight of the code. The size of $\mathcal{C}$ is the number s of its codeword-sets. The larger the size of the code is, the greater its usefulness.

Let $\Phi(v, k, \lambda_a, \lambda_c)$ be the largest possible size of a $(v, k, \lambda_a, \lambda_c)$ OOC. An OOC achieving this maximum size is said to be *optimal*. For codes with $\lambda_a = \lambda_c = 1$ we have the following upper bound [2]:

$$\Phi(v,k,1,1) \leq \left\lfloor \frac{v-1}{k(k-1)} \right\rfloor .$$

Consider a codeword-set $C = \{c_1, c_2, \ldots, c_k\}$. Denote by $\triangle' C$ the multiset of the values of the differences $c_i - c_j$, $i \neq j$, $i, j = 1, 2, \ldots, k$. Denote by $\triangle C$ the underlying set of $\triangle' C$. A $(v, k, \lambda_a, 1)$ OOC is *perfect* if $|\bigcup_{i=1}^{s} \Delta C_i| = v - 1$, that is if all nonzero differences are covered.

Theorem 1 *(proved in [3]) For a* $(v, 5, 2, 1)$ *OOC*

$$\Phi(v,5,2,1) \leq \begin{cases} \left\lfloor \frac{v}{12} \right\rfloor & for\ v \equiv 1\ (mod\ 132)\ or\ v \equiv 154\ (mod\ 924) \\ \left\lceil \frac{v}{12} \right\rceil & otherwise. \end{cases}$$

A $(v, 5, 2, 1)$-OOC is *optimal* when its size s reaches the upper bound of Theorem 1. The authors of [3] present one optimal $(v, 5, 2, 1)$-OOC for any length $v \leq 62$ when it exists. They also give many direct and recursive constructions for infinite classes of optimal $(v, 5, 2, 1)$-OOCs.

Definition 2 *Two* $(v, k, \lambda_a, \lambda_c)$ *optical orthogonal codes are multiplier equivalent if they can be obtained from one another by an automorphism of* Z_v *and replacement of codeword-sets by some of their translates.*

In [4] we classify up to multiplier equivalence all optimal $(v, 5, 2, 1)$ OOCs with $v \leq 104$. In this work we extend the classification with all optimal $(v, 5, 1)$ OOCs with $v \leq 89$. To do this we apply back-track search with minimality test on the partial solutions [5, section 7.1.2]. Our minimality test rejects the current partial solution if some of the automorphisms of Z_v can map it to a lexicographically smaller solution (that has already been constructed).

References

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