Construction of self-dual matrix codes

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Abstract. Matrix codes over a finite field $\mathbb{F}_q$ are linear codes defined as subspaces of the vector space of $m \times n$ matrices over $\mathbb{F}_q$. They are closely related to rank metric linear codes. In this paper, we show how to obtain self-dual matrix codes from a self-dual matrix code of smaller size using a method we call the building-up construction. We show that every self-dual matrix code can be constructed using this building-up construction. Using this, we classify, that is, we find a complete set of representatives for the equivalence classes of self-dual matrix codes of small sizes. In particular we have classifications for self-dual matrix codes of sizes 2×4, 2×5 over $\mathbb{F}_2$, of size 2×3, 2×4 over $\mathbb{F}_4$, of size 2×2, 2×3 over $\mathbb{F}_8$, and of size 2×2, 2×3 over $\mathbb{F}_{13}$, all of which have been left open from K. Morrison's classification.

We can define a generator matrix for matrix codes using the correspondence with linear block codes. Using this definition, we introduce the *building-up construction of self-dual matrix codes* and show that every self-dual matrix code is obtained this way. Thus, using this construction and the notion of equivalence for matrix codes given in [2], we have a new technique to classify self-dual matrix codes, different from what was done in [2] and add new results, as well. The classification is summarized on the table below.

Table 1: The number of inequivalent self-dual matrix codes of small sizes over the finite field $\mathbb{F}_q$ where $q = 2, 3, 4, 5, 8, 9, 13$. Values marked with $*$ and $**$ are the same values given in [2] and [1], respectively. Values in bold are new classifications which were previously unknown.

Size	$\mathbb{F}_2$	$\mathbb{F}_3$	$\mathbb{F}_4$	$\mathbb{F}_5$	$\mathbb{F}_8$	$\mathbb{F}_9$	$\mathbb{F}_{13}$
2×2	2^*	1^*	3^*	2^*	**5**	2^{**}	**2**
2×3	5^*		**5**	7^*	**5**	7^{**}	**7**
2×4	**20**	13^*	**36**	24^*			
2×5	**22**						
4×3	**442**						

References

[1] K. Morrison. *Equivalence and duality for rank-metric and matrix codes.* The University of Nebraska-Lincoln, 2012.

[2] K. Morrison. An enumeration of the equivalence classes of self-dual matrix codes. *Advances in Mathematics of Communications*, 9(4):415 – 436, May 2015.

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