Codes, matroids, and their q-analogues

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Abstract. Over the last decades, many have studied the relation between linear codes and matroids. One link between the two objects goes via the extended weight enumerator of a code and the Tutte polynomial of the corresponding matroid. The recent interest in network coding leads to the question if this link has a q-analogue. In this talk I will report on the ongoing quest for the q-analogues of matroids, the Tutte polynomial, and their link with network coding.

Greene was the first to notice that the weight enumerator of a linear code is defined by the Tutte polynomial of the matroid corresponding to the code. In my PhD thesis under supervision of Ruud Pellikaan, I studied a generalisation of the weight enumerator, the *extended weight enumerator*. This polynomial defines the Tutte polynomial of the corresponding matroid, leading to a two-way equivalence between the two polynomials.

In the last decade the focus in coding theory shifted to network coding, where communication is not over a single channel but over a network. Accelerated by the COST action "Random network coding and designs over $GF(q)$", codes with respect to the rank metric attracted a lot of attention. One can see these codes as a q-analogue of codes with respect to the Hamming metric. A q-analogue is, roughly speaking, what happens if we generalise from sets to subspaces.

Ruud and I started working on the q-analogue of the weight enumerator and its generalisations. The first results on the *rank weight enumerator* went very smoothly, and it seemed that a lot of results in classical coding theory were just waiting for a straightforward q-analogue. Quit soon we started fantasising about the q-analogues of matroids, the Tutte polynomial, and their link with the rank weight enumerator.

Unfortunately, q-analogues turned out not to be as straightforward as we initially thought. With this talk I hope to argue that this makes the topic in fact much more interesting. I will describe how we came to a sensible definition of the q-analogue of a matroid, how this links to earlier work of Henry Crapo, and what the next open problems in this subject are.

Joint work with Ruud Pellikaan (Eindhoven University of Technology) and Henry Crapo (Les Moutons Matheux, La Vaquerie).