Codes and Singularities

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For a given algebraic variety $\mathcal{X}$ over a finite field $\mathbb{F}$, a finite dimensional linear subspace L of $\mathbb{F}(\mathcal{X})$ and a finite set of rational P points of $\mathcal{X}$, the associated Evaluation Code $C = C(\mathcal{X}, L, P)$ is the image of the evaluation map of functions in L at the points in P. Algebraic geometry methods are classically available for studying those codes from the involved geometrical features. Their study becames specially succesful when the geometry of the data $\mathcal{X}, L, P$ allow to estimate the main parameters and give explicit expressions for their coding and decoding. This happens when $\mathcal{X}$ is a smooth projective curve or a toric variety among others.

Singularities are especially useful for some purposes. For instance, when the smooth curve $\mathcal{X}$ is the normalization of a singular proyective plane curve $\mathcal{Y}$, the parameters, coding and decoding of evaluation codes can be explicitly described by means of the classical Brill-Noether and Castelnuovo results. However, the evaluation is made on points P of $\mathcal{X}$ which are non singular points.

When the evaluation is made on singular points of algebraic varieties, the situation becomes even much more difficult. In fact, the complete information of the evaluation of a function at singular points includes an appropriated part of its Taylor expansion, which it is more than its single point evaluation. The main difficulty is that there are not general methods for estimating the minimal distance of the code, so one needs to find objects with really special geometric features.

Such special situation occurs when the set P is the singular subscheme of a foliations which isolated singularities of arbitrary degree r foliations by curves over the n-dimensional projective scheme over $\mathbb{F}$. Results by Campillo-Olivares show that, when $r > 2$, the foliation is determined by their singular subscheme and that P have very special geometric properties rather similar to those of the whole projective spaces. It allows to construct interesting examples of such P for many values of r inside a projective space of characteristic p, for many values of r and p. The evaluation codes of complete evaluation of polynomials of such P can be explicitly described and their parametes can be estimated. These result extend those proved by Campillo-Farran-Pisabarro for the special case of $n = 2$ and the singularities P reduced (say, counted one each of them).

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