Isometry-Dual Flags of AG Codes

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Abstract. Consider a complete flag $\{0\} = C_0 < C_1 < \cdots < C_n = \mathbb{F}^n$ of one-point AG codes of length n over the field $\mathbb{F}$. A flag has the isometry-dual property if the given flag and the corresponding flag of dual codes are the same up to an invertible diagonal transformation. In [2] it is shown, for a curve of genus g, that a complete flag of one-point AG codes defined with a set of $n > 2g+2$ rational points is isometry-dual if and only if the code C_n in the flag has Goppa divisor of degree $n + 2g - 1$. Using a different proof, we extend this characterization to all sets of size $n \geq 2g + 2$. Moreover we show that this is best possible by giving examples of isometry-dual flags with $n = 2g+1$ such that C_n has Goppa divisor of degree $n+2g-2$. We also prove a necessary condition, formulated in terms of maximum sparse ideals of a Weierstrass semigroup, under which a flag of punctured AG one-point codes inherits the isometry-dual property from the original unpunctured flag.

Let $\mathcal{X}$ be a smooth absolutely irreducible projective curve of genus g defined over the finite field $\mathbb{F}$. Let $P_1, \ldots, P_n$ and Q be distinct rational points on $\mathcal{X}$. For $D = P_1 + \cdots + P_n$, let $C_0 = C_L(D, -Q) = \{0\}$, and define a complete flag $\{0\} = C_0 < C_1 < \cdots < C_n = \mathbb{F}^n$ of one-point AG codes by choosing $m_1, \ldots, m_n$ minimal such that $C_i = C_L(D, m_i Q) \neq C_{i-1}$.

(Main Theorem) Let $m = m_n$. If the complete flag is isometry-dual then the following holds.

(a) If $m \geq 4g$, then $n = m - 2g + 1 \geq 2g + 1$.

(b) If $m = 4g - 1$, then either $n = 2g$ or $n = 2g + 1$.

(c) If $m \leq 4g - 2$, then $n \leq 2g$.

References

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