Classification of linear codes using canonical augmentation

Iliya Bouyukliev

Institute of Mathematics and Informatics,
Bulgarian Academy of Sciences, Veliko Tarnovo, Bulgaria

iliyab@math.bas.bg

Abstract. Classification of linear codes is an important task that affects the classification of other combinatorial structures such as finite geometries, combinatorial designs, etc. We propose a new algorithm for classification based on canonical augmentation.

The concept of canonical augmentation is introduced by Brandan McKey. It is a very powerful tool for classification of combinatorial structures. The canonical augmentation uses a canonical form to check the so called "parent test" and considers only objects that passed the test. Algorithms of this type have been used for classification of linear codes and before. The codes are represented by their generator matrix. To obtain generator matrices of all inequivalent codes of given length and dimension one begins from the empty set and constructs matrices column by column. In this way, to classify all linear $[n, k]$ codes, codes of the lengths $1, 2, \ldots, n$ and dimensions $\leq k$ are also constructed in the generation process.

We present a new algorithm of the same type but with a special modification which makes it much faster. Our algorithm also expands the matrices column by column but starts from the identity $k \times k$ matrix. So it constructs all inequivalent linear $[n, k]_q$ codes without getting codes of smaller dimensions. Restrictions on the dual distance, covering radius, minimal distance, etc. can be applied. The algorithm is included in the new version of the package Q-Extension. Using this software we classified and also proved the nonexistence of codes with given parameters over fields with 4, 5 and 7 elements. In this way, we solved some of the open problems presented in Code Tables: Bounds on the parameters of various types of codes: `http://www.codetables.de`.

The considered linear codes have parameters $[20+i, 13+i, 6]_4$, $[18+i, 7+i, 9]_4$, $[15+i, 4+i, 10]_5$, $[15+i, 5+i, 9]_5$, and $[14+i, 7+i, 7]_7$, $i \geq 0$. The result we obtain is that there are exactly two inequivalent $[20, 13, 6]_4$ codes, 10 inequivalent $[18, 7, 9]_4$ codes, 1628 inequivalent $[15, 4, 10]_5$ codes, and 4308 inequivalent $[15, 5, 9]_5$ codes. Further, we proved that codes with parameters $[21, 14, 6]_4$, $[19, 8, 9]_4$, $[16, 5, 10]_5$, $[16, 6, 9]_5$ and $[15, 8, 7]_7$ do not exist.

Joint work with Stefka Bouyuklieva (Faculty of Mathematics and Informatics, St. Cyril and St. Methodius University of Veliko Tarnovo, Bulgaria)
Partially funded by grant number DN 02/2/13.12.2016