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Curve Singularities and Hilbert Schemes

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SUMMARY

A famous formula of Strominger, Yau and Zaslow gives the "number" of rational curves C in a linear system on a $K3$ surface. According to Beauville, each of them has to be counted with the Euler number of the compactified jacobian as multiplicity. A result by Goetttsche, Fantechi and van Straten relates this to to the multiplicity of the δ-constant stratum of the plane curve singularities of C. Now, this turns out to be the tip of an iceberg. Recently, V. Shende related multiplicities of strata with fewer double points to Euler numbers of hilbert schemes and V. Shende and A. Oblomkov conjectured a relationship with the HOMFLY-polynomials of the knot associates to the curve singularity, which generalises the formula of Campillo, Delgado, and Gusein-Zade for the Alexander-polynomial. In the talk I will try to give an overview of these newer developments.

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