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Flat Singularity Theory

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SUMMARY

In a number of problems relating to singularities of plane curves it is interesting to take account of tangential singularities in the sense of points of inflexion etc. In a recent paper of Dias and Nuño, a definition of equivalence was given with this in mind, and a classification obtained for singularities of codimension at most 2. A further step was taken by Sinha and Tari, who gave a more convenient definition of equivalence, and detailed pictures of unfoldings of these cases, and of their duals, in situations arising from the family of orthogonal projections of a space curve. In these papers, there is no theory of versality properties of unfoldings, and my main motivation is to seek such a theory. I have attained only partial success in this.

There is a natural convenient definition of *flat equisingularity* by numerical, essentially topological invariants. This relation underlies the notation and the classification results obtained in the earlier papers.

There are many possible definitions of equivalence taking account of flatness. My preferred definition differs from both the earlier ones, but adapts more flexibly to singular points having more than one tangent direction, and admits a direct relation to the classical theory.

I have made no progress with a full analogue of the theory of versality, and do not believe this to be feasible. The notion of *F-transversality* parallels the classical criterion for versality, and I conjecture

(i) *F-transversality* is an open condition

(ii) Any two mini *F-transversal unfoldings* are equivalent in some sense.

I have a partial result for openness; as to uniqueness, I define a property which I can verify for a large number of cases, including most of low codimension.

I also classify all the singularities in the A-series and those in the D-series up to flat equivalence.

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