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Lax operator algebras and integrable systems

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SUMMARY

Lax operator algebras are introduced by I.Krichever and the author in 2006-2007. A key observation was as follows: the space of all matrix-valued Lax operators with a spectral parameter on a Riemann surface (I.Krichever, 2001) possesses a natural structure of an almost graded Lie algebra. It turned out to be true for the Lax operators taking values in a classic complex Lie algebra too. Let us notice that in the conventional theory of Lax equations the spectral parameter belongs to a rational curve, the corresponding Lax operator algebra descends to a Kac-Moody algebra and Riemann surfaces appear as spectral curves at the next step.

In the talk we are going to introduce Lax operator algebras and present the symplectic geometry and Hamiltonian theory of the corresponding Lax equations. It provides quite general approach to the Hamiltonian theory of many known and new integrable systems including Hitchin systems and their generalizations on punctured Riemann surfaces (in particular, Calogero-Moser systems), gyroscopes, integrable cases of motion of a rigid body in a fluid etc.

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