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Multi-variable Poincaré series, lattice cohomology and Seiberg-Witten invariants

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SUMMARY

Assume that Γ is a connected negative definite plumbing graph, and that the associated plumbed 3-manifold M is a rational homology sphere. We provide two new combinatorial formulae for the Seiberg-Witten invariant of M. The first one is the constant term of a 'multivariable Hilbert polynomial', it reflects in a conceptual way the structure of the graph Γ , and emphasizes the subtle parallelism between these topological invariants and the analytic invariants of normal surface singularities. The second formula realizes the Seiberg-Witten invariant as the normalized Euler characteristic of the lattice cohomology associated with Γ , supporting the conjectural connections between the Seiberg-Witten Floer homology, or the Heegaard-Floer homology, and the lattice cohomology.

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