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Geometric Theory of Parshin Residues

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SUMMARY

In the early 70's Parshin introduced his notion of the multidimensional residues of meromorphic top-forms on algebraic varieties. Parshin's theory is a generalization of the classical one-dimensional residue theory. The main difference between the Parshin's definition and the one-dimensional case is that in higher dimensions one computes the residue not at a point but at a complete flag of irreducible subvarieties $X = X_n \supset \ldots \supset X_0$, $\dim X_k = k$. Parshin, Beilinson, and Lomadze also proved the Reciprocity Law for residues: if one fixes all elements of the flag, except for X_k, where $0 < k < n$, and consider all possible choices of X_k, then only finitely many of these choices give non-zero residues, and the sum of these residues is zero. Parshin's constructions are completely algebraic. In fact, they work in very general settings, not only over complex numbers. However, in the complex case one would expect a more geometric variant of the theory. We study Parshin residues from the geometric point of view. In particular, the residue is expressed in terms of the integral over a smooth cycle. Parshin-Lomadze Reciprocity Law for residues in the complex case is proved via a homological relation on these cycles. In order to construct these cycles and prove the relation, we develop a theory of Leray coboundary operators for Whitney stratified spaces. These operators satisfy certain relations, which imply the Parshin's Reciprocity Law for residues in the complex case. This is a part of my PhD thesis under Professor Khovanskii supervision.

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