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Poincaré series of singularities: an overview

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SUMMARY

Over the last years Poincaré series have become a natural invariant of the topological type of complex singularities, mainly in low dimension (surface and curve singularities). The seminal work on Poincaré series was the computation of it in the case of plane curve singularities and the proof that the new object coincides with the Alexander polynomial of the corresponding link (Duke, 2003). The use of multi-index filtrations (defined from the natural valuations on the germ) and the interpretation of the coefficients of the series as the Euler characteristic of some natural spaces attached to the values of a function should be considered as (some of) the new ingredients involved.

Certainly a cornerstone in the development of Poincaré series was the construction and use of the integration on the ring of functions with respect to the Euler characteristic. The inspiration for this construction comes from two sources: firstly, the interpretation by Viro of the Euler characteristic as a measure and, secondly the development of the motivic integration by Konsevitch, Denef, Loeser, et al. The avatar of the Poincaré series as an integral w.r.t the Euler characteristic allows us to compute it also in other contexts (divisorial valuations, rational surface singularities, etc.) in all the cases in a shorter and more clear way than the one used before.

Recently the use of Poincaré series has been extended in other directions providing new interesting results, among others one can mention its coincidence with the Stöhr-zeta function (in the case of curves defined over finite fields), its extension to quasi-ordinary singularities, toric singularities, ...

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