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Generic singularities of averaged optimization of cyclic processes with discount

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SUMMARY

A cyclic process is modeled by a smooth control system on the circle with positive admissible velocities only and a control parameter belonging to a smooth closed manifold or a disjoint union of ones with at least two different points.

An *admissible motion* is defined as an absolutely continuous map x from a time interval to the circle such that at each moment of its differentiability the velocity $\dot{x}$ belongs to the convex hull of the admissible velocities of the system. A *cycle* with a period $T > 0$ is defined as a periodic admissible motion $x, x(t+T) \equiv x(t)$. In the applications there is usually a continuous *profit density* f, and the motion along the cycle collects the respective profit. That leads to the famous optimization problem: how to select a cycle providing the maximum of time averaged profit:

$$\int_0^T f(x(t))dt/T \to \text{máx}\,.$$

This problem was touched by various approaches. V.I.Arnold proposed the one based on the singularity theory achievements. He demonstrated that in a typical case the motion along an optimal cycle uses the maximum and minimum velocities when the profit density is less or greater than a certain constant, respectively [1], [2], [3] and analyzed some profit singularities. The classification of profit generic singularities was completed recently [3].

We generalized the theory for the case of cyclic processes in the presence of a discount rates. The talk is devoted to these results.

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Referencias

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