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DE INFORMÁTICA.
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UNIVERSIDAD DE VALLADOLID



Thematic Seminar on *Singularities, Algebraic Geometry, Computing and Information*

Universidad de Valladolid - E. U. Informática, Segovia

October 15-16th, 2009

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Presentation

The Thematic Seminar on *Singularities, Algebraic Geometry, Computing and Information* is thought as for a series of Workshops devoted to computational and applied topics related to Algebraic Geometry. The aim is to learn and discuss in depth about concrete topics which are of interest for mathematics community in Spain. Formally, the Seminar will be a two days Workshop devoted to Abstract or Computational Singularities, Algebraic Geometry and Information Theory. The workshop also is thought as a coordinating one of research projects developed by the organizing group SINGACOM.

This Seminar will be held at the Computer Science School in the Campus of Segovia of the Valladolid University within the dates October 15-16th (2009).

Participants are expected to arrive to Segovia in the afternoon of Wednesday 14th, and the Seminar will end in the evening on Friday 16th. Further details will be provided on the web page

<http://www.singacom.uva.es/oldsite/seminarios/WorkshopSG/workshop3/>.

Committees

Scientific Committee

- Antonio Campillo López (Valladolid University)
- Félix Delgado de la Mata (Valladolid University)
- Santiago Encinas Carrión (Valladolid University)

Organizing Committee

- José Ignacio Farrán Martín (Valladolid University)
- Carlos Galindo Pastor (Jaume I Castellón University), Secretary.
- Evelia García Barroso (La Laguna University).

List of participants

- Argimiro Arratia Quesada (UPC, Barcelona)
- Rocío Blanco Somolinos (Castilla-La Mancha University)
- Antonio Campillo López (Valladolid University)
- Julio Castellanos Peñuela (UCM, Madrid)
- Félix Delgado de la Mata (Valladolid University)
- Eugenia Ellis Raggio (Buenos Aires University)
- Santiago Encinas Carrión (Valladolid University)
- José Ignacio Farrán Martín (Valladolid University, Segovia)
- Ana Belén de Felipe Paramio (La Laguna University)
- Rosa M. de Frutos Marín (Valladolid University)
- Carlos Galindo Pastor (Jaume I University)
- Evelia García Barroso (La Laguna University)
- Fernando Hernando Carrillo (University College, Cork, Ireland)
- Mustapha Lahyane (Universidad Michoacana, Morelia, México)
- Jaime Lugo (Valladolid University)
- Carlos Marijuán López (Valladolid University)
- Irene Márquez Corbella (Valladolid University)
- Francisco Monserrat Delpalillo (UPV, Valencia)
- Carlos Munuera Gómez (Valladolid University)
- C. Ana Núñez Jiménez (Valladolid University)
- Jorge Olivares Vázquez (CIMAT, Guanajuato, México)
- María Jesús Pisabarro Manteca (Leon University)

- Brenda Leticia de la Rosa Navarro (Universidad Michoacana, Morelia, México).
- Alberto Vigneron-Tenorio (Cádiz University)

Schedule

	Thursday15th	Friday16th
9:30-10:15	Jorge Olivares	Evelia García
10:15-11:00	Mustapha Lahyane	Francisco Monserrat
11:00-11:30	<i>Coffee Break</i>	<i>Coffee Break</i>
11:30-12:15	Eugenia Ellis	María Jesús Pisabarro
12:15-13:00	Antonio Campillo	Rocío Blanco
13:00-13:45	Félix Delgado	Alberto Vigneron
13:45-16:00	Lunch	Lunch
16:00-16:45	Irene Márquez	Fernando Hernando
16:45-17:15	Carlos Marijuán	Carlos Galindo
17:15-17:30	<i>Break</i>	<i>Break</i>
17:30-18:15	Argimiro Arratia	Rosa de Frutos
18:15-19:00	Julio Castellanos	Santiago Encinas

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Abstracts

1.1 Ranking pages and the topology of the web

Argimiro Arratia (Univ. Polit. Catalunya)

Our work deals with ways to rearrange the link structure of websites for improving the PageRank of a page, or group of pages. (PageRank is Google ranking function for web pages.) Here, we begin our topological taxonomy analysing trees with either unidirectional or bidirectional edges. For both cases we show that to compute the PageRank of the root in directed rooted trees, it is sufficient to count the number of nodes per level. From these results we derive closed formulas for the PageRank of the root of various types of trees, establish a hierarchy of these topologies in terms of PageRank and give rules for modifying the links in these tree-like websites that increase their PageRank. (This is joint work with Carlos Marijuán, UVa)

1.2 Resbin.lib a new library of resolution of singularities to SINGULAR

Rocío Blanco (Univ. Castilla-La Mancha)

This talk will be an overview of “resbin.lib” (RESolution of BINomial ideals). This library is a joint work with Gerhard Pfister, from Technical University Kaiserslautern.

This new library allows to compute explicitly a resolution of singularities of a binomial ideal over a field of arbitrary characteristic. The theoretical algorithm of resolution of singularities implemented in this library appears in [Bla09].

References

- [Bla09] Combinatorial resolution of binomial ideals in arbitrary characteristic. Preprint. arXiv:0902.2887v1 [math.AG].
- [GPS09] G.-M. Greuel, G. Pfister, and H. Schönemann. Singular 3.0.4. A computer algebra system for polynomial computations. <http://www.singular.uni-kl.de> (2007).

1.3 Degree and regularity of invariant varieties by foliations on projective spaces

Antonio Campillo (Univ. Valladolid)

This is a survey talk on bounding degree and regularity of s -dimensional projective varieties which are invariant by s -dimensional foliations on P^n , $s < n$, as well as applications of the obtained bounds in practice.

1.4 Nash blowing ups of curves

Julio Castellanos (Univ. Complutense de Madrid)

We study the properties of the curves (space or plane) obtained by Nash blowing up of curves. We suggest that they can be useful for moduli problems.

1.5 On the topology of the image by a morphism of plane curve singularities

Félix Delgado (Univ. Valladolid)

For a finite morphism from the plane to the plane we describe the topology of the image of a branch in the source by the use of iterated pencils of analytic functions, constructed inductively in a natural way starting from the components of the map. In particular the topology (expressed by the maximal contact values) of an irreducible germ could be described. The relations between branches of the critical locus of the map and the special fibres of the pencil defined by the components of the map allows to determine the topological type for all the branches of the discriminant curve. (In collaboration with H. Maugeudre).

1.6 Equivariant algebraic kk -theory and adjoints theorems

Eugenia Ellis (Univ. Buenos Aires)

We shall see that algebraic kk -theory defined by G. Cortiñas and A. Thom admits an equivariant version. If k is a field and G is a group we define equivariant algebraic kk -theory for kG -algebras. Also, we defined an equivariant algebraic kk -theory for H -module algebra where H is a finite dimensional Hopf algebra. We show algebraic versions of some theorems in the context of operator algebras and Kasparov KK -theory: Green-Julg Theorem, Green imprimitivity Theorem and Baa-j-Skandalis duality. This work is joint with G. Cortiñas.

1.7 Łojasiewicz exponents, resolution of singularities, algorithms and quasi-homogeneous polynomials

Santiago Encinas (Univ. Valladolid)

This talk is intended to show some results for computing Łojasiewicz exponents which have been recently developed with C. Bivià-Ausina.

Using implemented algorithms of resolutions of singularities, one would be able to compute the Łojasiewicz exponent of any ideal. However complexity problems appear.

On the other hand, there is a conjecture on the value of the Łojasiewicz exponent for the gradient of a quasi-homogeneous polynomial with isolated singularity. Recently Krasiński, Oleksik and Płoski have proved the conjecture in the case of three variables. We will show a result on the Łojasiewicz exponent of quasi-homogeneous ideals. Assuming some additional conditions on the degree and the weights of the quasi-homogeneous polynomial, we prove the conjecture as a consequence of the previous result. The assumptions are satisfied if degree is a multiple of all weights.

1.8 Polynomials sharing roots with its derivatives

Rosa de Frutos (Univ. Valladolid)

Conditions of having a common root with derivatives on polynomials can be formulated as schemes over the ring of integers and they provide a relation between algebra and number theory which give rise to some progress on concrete problems in the context.

1.9 Plane valuations at infinity. Applications to coding theory

Carlos Galindo (Univ. Jaume I)

In the talk, I will introduce the concept of plane valuation at infinity. These valuations are defined by suitable (infinite) families of plane curves having only one place at infinity and one can consider for them a semi-group at infinity. I will also introduce generalized delta-sequences and I will show that an analogue of the Abhyankar-Moh semigroup theorem holds for plane valuations at infinity. In addition, it will be explained how these valuations allow to construct a large class of evaluation codes, containing the Reed-Solomon ones, that can be decoded by the Berlekamp-Massey-Sakata algorithm. This research has been gone on with F. Monserrat.

1.10 A discriminant criterion of irreducibility

Evelia García-Barroso (Univ. La Laguna)

In [1] we give criteria of irreducibility for a complex power series in two variables, using the notion of jacobian Newton diagrams, defined with respect to a generic direction. In this talk we generalize these criteria to any direction and we use this new general criterion to study the branches at infinity of polynomial curves. This is a joint work with Janusz Gwoździewicz. The talk is based in the results obtained in [2].

References

- [1] E. García Barroso-J. Gwoździewicz, *Characterization of jacobian Newton polygons of plane branches and new criteria of irreducibility*. To appear in Annales de l'Institut Fourier.
- [2] E. García Barroso-J. Gwoździewicz, *A discriminant criterion of irreducibility*. Preprint.

1.11 BCH codes as evaluation codes

Fernando Hernando (Cork Univ.)

BCH codes constitute one of the most used families in coding theory because of the efficient encoding and decoding algorithm. These codes may be defined as the zeros of the codewords defining them. The aim of this work is to describe from another perspective, that is, in terms of the so called evaluation codes. We will also want to generalize this interpretations and techniques to another families of codes.

1.12 On the Finite Generation of the Effective Monoid of Rational Surfaces

Mustapha Lahyane (Univ. Michoacana, Morelia, México)

The aim is to give a new class of rational surfaces for which the effective monoid is finitely generated. In particular, we show that the singularities do not have an impact on the finite generation.

1.13 Connective structure and cyclic structure of a digraph

Carlos Marijuán (Univ. Valladolid)

Some inverse spectral problems require the connective structure and the cyclic structure of a digraph to be studied but this can be reduced to the study of the order structure of an acyclic digraph and the cyclic structure of a strong digraph. We give characterizations of the order structure of an acyclic digraph. We

propose the construction of strong digraphs from minimal strong digraphs and we give sequentially generative procedures for the constructive characterization of the classes of minimal strong digraphs and strong digraphs. We look at some natural spectral and enumerative properties in this context. Finally, we propose a matrix translation of this theory for the spectral characterization of classes of Boolean matrices and we describe some open problems.

1.14 Security of stream ciphers against fault attacks

Irene Márquez (Univ. Valladolid)

SNOW3G is the back up encryption algorithm of the mobile phone UMTS technology, used to keep data confidential. Its design -a combiner with memory- is derived from the stream cipher SNOW2.0, with improvements against algebraic cryptanalysis and distinguishing attacks. No attack is known against SNOW3G today. In this talk, a fault attack against SNOW3G is proposed requiring 22 fault injections and less than one minute of off-line computation. We also propose a realistic countermeasure to protect SNOW3G with very low additive cost.

1.15 Linear systems of plane curves and foliations of the projective plane

Francisco Monserrat (Univ. Polit. Valencia)

Some results related with the Harbourne-Hirschowitz Conjecture on linear systems of plane curves will be given. A consequence of this conjecture is a prediction on the structure of the Mori cone of the surface obtained by blowing-up a finite set of points of the projective plane in very general position. More specifically, we shall show a result which provides evidences of this structure and whose proof involves a family of algebraic foliations given by A. Lins Neto.

1.16 Special subschemes of the (sub-)scheme of singularities of a plane foliation.

Jorge Olivares (CIMAT, México)

Let \mathcal{F} be a foliation of degree $r \geq 2$ in the complex projective plane \mathbb{P}^2 , with isolated singularities $S(\mathcal{F})$. It is well-known ([Gómez-Mont,Kempf], [Campillo,-]) that the map $\mathcal{F} \mapsto S(\mathcal{F})$ is injective (that is, that such an \mathcal{F} is uniquely determined by the subscheme of its zeroes). We pose the following problems:

1. The problem of existence of proper subschemes $Z \subset S(\mathcal{F})$ such that still \mathcal{F} is uniquely determined by Z , and
2. In case of an affirmative answer to problem 1, to give geometric characterizations of such subschemes Z , to compute its minimal possible degree $m = m(r)$ and to solve problem 1 for such subschemes of degree m .

In the talk, we will compute the value of $m(r)$ for $r \geq 2$, we will show that problem 1 always has a solution Z , for $r \geq 4$, but that $\deg Z > m(r)$. We shall discuss our advances toward the general solution of problem 2 (this a joint work-in-progress with A. Campillo).

1.17 Systems of generators of monomial curves

María Jesús Pisabarro (Univ. León)

It is known the relationship between irreducible curves with monomial parametrization and binomial prime ideals. The exponents of the parametrization of an irreducible monomial curve generate a semigroup S , cancelative and combinatorially finite, such that the ideal of the curve is S -homogeneous. This S -graded structure that satisfies Nakayama's lemma provides combinatorial methods for computing a minimal S -graded free resolution for the ring of the curve, and in particular to calculate minimal systems of generators. Pisabarro's thesis gives a generalization of the monomial curve for reduced case: essentially a curve with binomial ideal whose irreducible components are monomials. Using the monomial parametrizations of the components, we can construct a semigroup S such that the ideal of the monomial curve is S -homogeneous. Unlike the irreducible case, this semigroup S is not cancelative nor combinatorially finite, and, in fact, we can find minimal systems of generators with different number of elements. However, this graduation satisfies a "partial" Nakayama's lemma which makes us to think about combinatorial methods for computing minimal systems of generators for monomial curves. In this talk, I will explain the S -graded structure for monomial curves, and the way we are trying to find systems of generators.

1.18 Markov bases in Algebraic Statistics

Alberto Vigneron-Tenorio (Univ. Cádiz)

The Markov bases (introduced in [2]) let construct Markov chain from contingency tables. The problem about the existence or non-existence of a unique Markov bases seems to be interesting for Statisticians (see e.g. [1],[3]), and it is closely related to the computation on toric ideals. We study that problem by using Computational Algebra and Combinatorics (see [3]).

References

- [1] S. Aoki, A. Takemura, R. Yoshida. *Indispensable monomials of toric ideals and Markov bases*. Journal of Symbolic Computation, 43 (2008), 490–507.
- [2] P. Diaconis, B. Sturmfels. *Algebraic algorithms for sampling from conditional distributions*. Ann. Statist. 26(1) (1998), 363–397.
- [3] A. Takemura, S. Aoki. *Some characterizations of minimal Markov basis for sampling from discrete conditional distributions*. Ann. Inst. Statist. Math. 56(1) (2004), 1–17.