SECOND WORKSHOP ON ZETA FUNCTIONS IN ALGEBRA AND GEOMETRY.

PALMA DE MALLORCA, May 3-7, 2010.

ABSTRACTS

Yuri Manin

"Modular symbols for Maass wave forms and Levy-Mellin transform"

Abstract: In this talk I extend the notion of modular symbols to Maass wave cusp forms. They appear in the guise of finitely additive functions on the Boolean algebra generated by intervals with non--positive rational ends, with values in analytic functions (pseudo--measures in the sense of Manin-Marcolli). I will also discuss the notion of L\'evy--Mellin transform and argue that it provides an \infty-adic analog of the p-adic Mazur-Mellin transform, which itself was constructed as an analog of the classical Mellin transform.

Yuri Tschinkel

"Igusa integrals and volume asymptotics in analytic and adelic geometry" (joint work with A. Chambert-Loir).

Abstract: I will explain how to estimate volumes of height balls in analytic varieties over local fields and in adelic points of algebraic varieties over number fields, relating the Mellin transforms of height functions to Igusa integrals and to global geometric invariants of the underlying variety. In the adelic setting, this involves the construction of general Tamagawa measures.

Pilar Bayer

"Computational aspects of Artin L-functions"

Abstract: For weight $k \geq 2$, the dimension of the complex vector space $S_k(N, \chi)$ of modular cusps forms of type (N, k, χ) can be computed by using either Riemann-Roch theorem or Selberg-Eichler trace formula. When $k=1$ no such formulas are know, although asymptotic formulas for $\dim(S_1^{new}(N, \chi))$ have been conjectured. The main term in these formulas should be provided by the cups forms of dihedral type (namely, theta functions); the error term, for the rest of the forms, called exotic:

$$\dim (S_1^{new}(N, \chi)) = s^{Dih} (N, \chi) + s^{Exotic} (N, \chi) .$$

For N prime and character equal to the Legendre symbol, Duke obtained in 1995 that $s^{Exotic}(N, \chi) << N^{1-\delta}$ for any $0 \leq \delta < 1/12$. In 1975, Deligne and Serre had proven that the L-series of new modular cusps forms of weight one agree with the Artin L-series of odd irreducible complex Galois representations of dimension 2. In our talk, we shall explain how Deligne-Serre theorem and explicit solutions of Galois embedding problems (due to Crespo, Quer, *et al*.) can be used to determine the Fourier coefficients of exotic modular forms of weight one.

Jan Schepers

"Stringy Hodge numbers of strictly canonical nondegenerate singularities"

Abstract: Motivated by examples of singular mirror dual Calabi-Yau varieties from string theory, Batyrev defined in 1997 the stringy E-function for complex algebraic varieties with certain `mild' singularities. It is a rational function in two variables. If the variety Y is projective and if the stringy E-function is a polynomial, then Batyrev defined the stringy Hodge numbers of Y, essentially as the coefficients of this polynomial. They satisfy analogous properties as usual Hodge numbers of smooth projective varieties and coincide with them for smooth Y. However, for singular Y it is from the definition not at all clear that they are nonnegative. This was conjectured by Batyrev. In this talk we first explain how to obtain the stringy E-function of a hypersurface from the motivic zeta function of Denef and Loeser, and how to compute the stringy E-function of a nondegenerate hypersurface singularity. Then we describe a class of strictly canonical nondegenerate singularities that give rise to a polynomial stringy E-function. Moreover, in many cases we can prove Batyrev's conjecture for projective varieties with such singularities using combinatorics of lattice polytopes.

Félix Delgado de la Mata

"On Poincaré series of a finite number of plane valuations"

Abstract: Recently we have computed the Poincaré series of a finite number of valuations on the complex plane centered at the origin. The formula generalizes previous results for collections of valuations corresponding to curve singularities and for divisorial valuations. We prove also that the Poincaré series determines the topology of the collection. The results are included in a joint paper with S.M. Gusein-Zade, A. Campillo and F.Hernando.

Wolfgang Ebeling

Multi-variable Poincaré series associated with Newton diagrams

Abstract: This is a report on joint work with S.M.Gusein-Zade. We define a multi-index filtration on the ring of germs of functions on a hypersurface singularity associated with its Newton diagram and compute the multivariable Poincaré series of this filtration in some cases. We also discuss the relation with a Poincaré series of embedded filtrations defined by A. Lemahieu.

Alex Lubotzky

"Zeta functions counting representations of arithmetic groups"

Abstract: Let G be an higher rank arithmetic group. By Margulis super-rigidity a_n= the number of irreducible complex representations of G of degree n is finite. Let Z_G(s)= SUM (a_n n^(-s)) be the associated "zeta function". We will describe some basic results on this function (a kind of Euler factorization, abssisca of convergence etc.). Based on a joint work with Larsen and some work of Avni.

Pierrette Cassou-Noguès

“Newton trees associated to ideals”

Abstract: This is a common work with W. Veys. In this talk we will explain how to define Newton trees associated to ideals of C[[x,y]] using the Newton process. For the moment, we can associate the vertices of those trees to the poles of the topological Zeta function of to the ideal. The hope is that we can calculate other invariants of the ideal using the Newton tree.

Antonio Rojas-León

“Moment L functions for a family exponential sums”

Abstract: The moment L-functions of a one parameter family of exponential sums are a sequence of rational functions that give a certain measure of the distribution of the sums along the family. In this talk we will show how to compute explicitely the trivial factors of the moment L-functions of a certain family. As an application, we give a sharp bound for the number of rational points of some Artin-Schreier curves. This is joint work with D. Wan.

Christopher Voll

"Representation zeta functions of p-adic analytic and arithmetic groups: p-adic formalism"

Abstract: In my talk I will report on aspects of joint work with N. Avni, B. Klopsch and U. Onn. Representation zeta functions are Dirichlet generating functions enumerating the finite-dimensional irreducible complex representations of infinite groups. For several classes of groups, these zeta functions are known to have interesting arithmetic properties. The representation zeta function of an arithmetic group with the Congruence Subgroup Property, for instance, has an Euler product into local factors enumerating representations of various completions of the group. The non-archimedean Euler factors are zeta functions of compact p-adic analytic groups. I will discuss how a Kirillov orbit formalism for these groups allows us to study their representation zeta functions with methods from p-adic integration.

This allows us to prove, in particular, local functional equations and, in the case of "semisimple" p-adic analytic groups, to establish connections with algebraic Lie theory. Our methodology is sufficiently powerful to allow for computations of representation zeta functions associated to the groups SL_3 and SU_3. These computations play a crucial role in our proof of a special case of a conjecture of Larsen and Lubotzky.

Nero Budur

“The monodromy conjecture for hyperplane arrangements”

Abstract: We present recent results on the monodromy conjecture of Igusa-Denef-Loeser for zeta functions of hyperplane arrangements. This is joint work with M. Mustata, M. Saito, Z. Teitler, and S. Yuzvinksy.

Takashi Taniguchi

“Orbital L-functions for the space of binary cubic forms”

Abstract: The zeta function for the space of binary cubic forms was introduced and studied by Shintani as an example of zeta functions for PV (Prehomogeneous Vector spaces). In this talk we define an orbital L-function for the space, and study its analytic properties. The construction is valid for some other PVs. We also discuss its applications. In particular, we will give an example of the zeta function which satisfies an analogue of Ohno-Nakagawa relation.

Dorian Golfeld

“Multiple Dirichlet series”

Abstract: Multiple Dirichlet series are zeta functions in several complex variables that have meromorphic continuation in all the variables with finitely many poles and satisfy a finite group of functional equations. We shall discuss some new constructions of multiple Dirichlet series coming from the theory of automorphic forms.

Jordan Ellenberg

"Zeta functions for enumeration of rational points and field extensions over function fields: a topological approach"

Abstract: The problem of counting extensions of a global field of discriminant at most X has been much studied, by e.g. Linnik, Davenport-Heilbronn, Malle, Roberts, and Bhargava. This counting problem can be presented in terms of a zeta function that sums (Disc L)^{-s} over all extensions L/K with some fixed degree (or, more generally, with some fixed Galois group, some fixed ramification behavior at specified primes, etc...) Similarly, the problem of counting rational points of height at most B on a variety X/K can be presented in terms of a "height zeta function" that sums (Height(P))^{-s}, as P ranges over X(K).

I will discuss joint work with Akshay Venkatesh and Craig Westerland which tells us something about the behavior of these zeta functions in case K is a rational function field over a finite field. I will concentrate mainly on the case of counting field extensions with dihedral Galois groups, which is closely related to the Cohen-Lenstra conjectures. In particular, I will explain how to prove a weak version of these conjectures for K = F_q(T) as a consequence of a topological theorem about homological stabilization for moduli spaces.

Ann Lemahieu

“Poincaré series of embedded filtrations”.

Abstract: We define a Poincaré series for a subspace of a complex analytic germ, induced by a multi-index filtration on the ambient space. We compute this Poincaré series for subspaces defined by principal ideals. For plane curve singularities and nondegenerate singularities this Poincaré series yields topological and geometric information. We compare this Poincaré series with the one recently introduced by W. Ebeling and S.M. Gusein-Zade. Finally we will talk

about some generalisations of the notion of Poincaré series, which is work in progress with Antonio Campillo.

Johannes Nicaise

"A proof of the motivic monodromy conjecture for abelian varieties"

Abstract: We formulate a global form of Denef and Loeser's motivic monodromy conjecture, and we prove it for tamely ramified abelian varieties A over a discretely valued field. More precisely, we show that the motivic zeta function of A has a unique pole, which coincides with Chai's base change conductor c(A), and that this pole corresponds to a monodromy eigenvalue on the tame ell-adic cohomology of A. This is joint work with Lars Halvard Halle (Hannover).

Benjamin Klopsch

"Representation zeta functions of arithmetic groups of type A2"

Abstract: A group G is said to be (representation) rigid, if it admits only finitely many irreducible complex representations of any given finite dimension. The representation zeta function of a rigid group G is the Dirichlet generating function which enumerates the finite-dimensional irreducible complex representations of G. By a theorem of Lubotzky and Martin, arithmetic lattices in semisimple groups with the Congruence Subgroup Property are rigid. A conjecture of Larsen and Lubotzky asserts that, for every higher rank semisimple group H the abscissa of convergence of the representation zeta function of any irreducible lattice in H is already determined by the ambient group H. I will report on joint work with N. Avni, U. Onn, and C. Voll on representation zeta functions of compact p-adic analytic groups and arithmetic groups. The emphasis of my talk will be on how our results lead to a proof of the conjecture of Larsen and Lubotzky in a special case, namely for arithmetic lattices in simple groups of type A2 over number fields

Aner Shalev

Some zeta functions and their applications to groups and probability.

Abstract:

Takashi Taniguchi

"Orbital L-functions for the space of binary cubic forms"

Abstract: The zeta function for the space of binary cubic forms was introduced and studied by Shintani as an example of zeta functions for PV (Prehomogeneous Vector spaces). In this talk we define an orbital L-function for the space, and study its analytic properties. The construction is valid for some other PVs. We also discuss its applications. In particular, we will give an example of the zeta function which satisfies an analogue of Ohno-Nakagawa relation.

Josep Domingo-Ferrer

“The zeta function and data mining”

Abstract: This talk will survey the connections between the Riemann zeta function and data mining. In fact, there is a synergy: the zeta function is used in data mining and data mining is used to study the zeta function. Zipf's law (1949), also known as zeta distribution, is a case of rank-size distribution and a well-known application of the zeta function to data analysis. The law states that, out of a population of data items ranked by decreasing frequency of appearance, the frequency of the item of any rank can be expressed in terms of the zeta function. Zipf's law is generalized by the Zipf-Mandelbrot law (1965). Robson's zeta theory (2005, 2008) has recently been proposed to support exhaustive mining of association rules in high-dimensional datasets. The idea is to represent the possible values of each attribute in a record as prime numbers. In that way, each record is represented by a composite number whose unique factorization yields the values the various attributes take in that record. The zeta function is used to measure the information carried by the joint appearance of two attribute values. If a joint appearance is very surprising (it appears much more or much less often than expected), then seeing it entails a lot of information, and conversely. The other side of the coin, that is, the use of data mining to study the zeta function is illustrated by Muqattash (2004). The embedded patterns and relations among the imaginary parts of the non-trivial zeroes of the zeta function are analyzed using data mining techniques, in an attempt to simplify the validation of the Riemann Hypothesis. This talk is with the assistance by Klara Stokes.

Anne Frubis-Krüger

'Some new features in Singular'

Abstract: The computer algebra system Singular is well known for its wide range of implemented algorithms, including some now considered basic tools like normalization and primary decomposition, and more sophisticated ones like (several variants of) desingularization in characteristic zero and many more. Central to most of them is the fast Groebner Bases engine in polynomial rings over fields, in the commutative and non-commutative setting. Only recently, this key feature has been extended to the case of polynomial rings over the integers and over rings of type Z/nZ opening up a whole new field of applications. The very first of these already exist in prototype implementations. In this talk, I shall focus on these new developments in Singular, their background and the current state of implementations

Evgeny Gorsky

“Motivic Poincare series and knot homology”

Abstract: The notion of the motivic Poincare series of a multi-index filtration on the ring of germs of functions was introduced by A. Campillo, F. Delgado and S. Gusein-Zade as a natural deformation of the Poincare series of such a filtration. Thr explicit algorithm for the computation of the motivic Poincare series will be presented. Some natural properties of this multi-variable power series, its rationality and symmetry property follow from this algorithm.

It turns out that the structure of the motivic Poincare series and its properties are similar to the ones of the Heegard-Floer knot homology theory proposed by P.Ozsvath and Z.Szabo. For irreducible germs the direct relation between these theories can be proved.

Driss Essouabri

"On (multiple) zeta functions associated to arithmetic functions of several variables"

Abstract: The multivariable arithmetic functions play an important role in several branches of mathematics. Depending on context, several types of zeta functions can be associated to them. The purpose of this talk is to give some results on analytic continuation of certain classes of these zeta functions. In particular, we will attach to any multiplicative multivariable function f: N^n --- C a Newton polyhedron and explain how this polyhedron allows to control the analytic continuation (i.e. polar divisors, their multiplicities, etc. ..) of its zeta functions and therefore gives information on the densities functions attached to the original arithmetic or geometric function f. As application, we will give some results on counting functions of rational points of algebraic varieties and the height zeta functions attached to them. We will also give some results on multiple Igusa functions associated to groups and to rings.

Pedro González-Pérez

"On the rational form of certain motivic Poincaré series".

Abstract: The geometric and arithmetic motivic Poincaré series of an algebraic variety where introduced by Denef and Loeser inspired by some classical Poincaré series in arithmetic geometry. Denef and Loeser proved that these motivic series have rational forms, but unlike motivic zeta functions there is no general expression for them in terms of a resolution of singularities. The rational form of these motivic series have been shown explicitly for a very few classes of singular varieties. In this talk we describe the rational form of these motivic series in the case of toric varieties in terms of the combinatorial convexity properties of the Newton polyhedra of certain monomial ideals.

Francois Loeser

"Local densities for p-adic sets"

Abstract: Using a regularization device, it is possible to define local densities for p-adic definable sets. These local densities may be computed on tangent cones and using projections. Our most advanced results rely on the following fundamental theorem: "a definable p-adic function which is locally Lipschitz continuous with some constant is piecewise globally on each piece Lipschitz continuous with possibly some other constant", which can be seen as a substitute for the Mean Value Theorem over the p-adics. This is joint work with Cluckers and Comte.

Shouwu Zhang

"Linear forms, algebraic cycles, and derivatives of L-series2.

Abstract: In this talk, I will state some conjectures and examples concerning the central derivatives of L-series in terms of invariant linear forms on autormorphic representations with

inner products defined by integrations of matrix coefficients, and algebraic cycles on Shimura varieties with Beilinson--Bloch height pairings.

Uri Onn

“Representation zeta functions and conjugacy class zeta functions”

Abstract: Let O be the ring of integers in a number field k. Let O_v be the v-adic completion of O with respect to a finite place v of k. In this talk I will describe a methodology which allows the explicit computation of the representation zeta functions of the groups SL_3(O_v). This computation is used to derive the asymptotic behavior of the (finite dimensional) representations of the group SL_3(O) [joint work with N. Avni, B. Klopsch and C. Voll]. I will also discuss some results in positive characteristic which indicate that the zeta function attached to G(O_v), for a ‘nice’ algebraic group G, depend merely on the size of the residue field and not on the structure of the ring O_v. This phenomenon is demonstrated also for the generating function which counts the number of representations of given level, which amounts to counting conjugacy classes in the congruence quotients of the group [joint work with M. Berman and P. Paajanen].

Francisco Monserrat

“The Poincaré series of multiplier ideals of a simple complete ideal”

Abstract: For a simple complete ideal of a local ring at a closed point on a smooth complex algebraic surface, it will be considered an algebraic object, named Poincaré series, that gathers in a unified way the jumping numbers and the dimensions of the vector space quotients given by consecutive multiplier ideals attached to the simple ideal. We will show that this series is "rational'', giving an explicit expression for it.

Andras Nemethi

Multi-variable Poincaré series and Seiberg-Witten invariants

Abstract: We will discuss different aspects regarding the topological multi-variable zeta function associated with the resolution graph of a normal surface singularity. These objects might serve as the source of deep analytic information (via generalizations of Campillo--Delgado--Gusein-Zade type identities, e.g. for splice-quotient singularities), but it also provides the Seiberg-Witten invariants of the link. We explain also the bridge connecting the Campillo--Delgado--Gusein-Zade identity and the Seiberg-Witten Invariant Conjecture.