Overview of the method

Let $\mathcal{A}$ be a finitely generated algebra (we want to solve a problem in $\mathcal{A}$).

- To find the appropiate morphism

$$\xi : K[X] \to \mathcal{A}.$$

  $\mathcal{I}$ will be the ideal such that

$$\mathcal{A} \cong K[X]/\mathcal{I} \cong Span_K(N) \text{ } (N\text{: the set of canonical forms}).$$

  If $\mathcal{A}$ is a monoid (or group) algebra ($A = K[M]$)

$$\mathcal{I} = \langle \{x^w - x^v \mid \xi(x^w) = \xi(x^v),\ x^w, x^v \in [X]\}\rangle.$$

- To find or construct an appropiate ordering $\prec$ on $[X]$.

- Define a reduction process (s.t. it allows to solve the problem):
  - $\star$ finite numbers of reductions,
  - $\star$ unique canonical forms.

Overview of the method: the instance of linear codes

Having:
$\xi : K[X] \to \mathcal{A} = K[M]$,
$K[M] \cong K[X]/\mathcal{I} \cong Span_K(N)$ (N: the set of canonical forms)

The instance of linear codes:

1. $M = \mathbb{F}_2^{n-k}$ (the monoids of the syndromes).
2. ξ: gives the syndrome of each $x^w \in [X]$.
3. $N \Leftrightarrow$ the coset leaders.
4. $\mathcal{I}$: we call it the ideal associated with the code.

Then, we compute a Gröbner basis of $\mathcal{I}$ for a convenient ordering $\prec$ such that:

$$Can(x^w, I, \prec) \Leftrightarrow \text{the coset leader with syndrome } \xi(x^w).$$