RAMANUJAN-ORR TYPE SERIES

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Abstract. The usual proofs of the Ramanujan-type series for $1/\pi$ are based on the use of elliptic integrals and modular equations. A recent method (2014), simplifies this approach by avoiding the need for singular values of the second kind. In addition this new technique allows to prove a wider class of series for $1/\pi$, and in this talk we will derive the following example:

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{8}\right)_n \left(\frac{3}{8}\right)_n \left(\frac{5}{8}\right)_n \left(\frac{7}{8}\right)_n}{\left(\frac{1}{2}\right)_n (1)_n^3} \left(\frac{192}{2401}\right)^n \frac{376n^2+216n+15}{2n+1} = \frac{98\sqrt{21}}{9\pi},$$

where $(a)_n = a(a+1)\cdots(a+n-1)$ and $(a)_0 = 1$. We will say that this is an example of a *Ramanujan-Orr series* for $1/\pi$, because in the proof of it we use an Orr-type factorization. Also, in this talk we will explain how to discover and prove several other results. For example

$$\sum_{n=0}^{\infty} \frac{\left(\frac{1}{2}\right)_n^5}{(1)_n^5} \frac{(-1)^n}{2^{10n}} (820n^2+180n+13) = \frac{128}{\pi^2},$$

is of similar aspect to those series by Ramanujan for the constant $1/\pi$, but it belongs to a new family of series for the constant $1/\pi^2$. We proved it in 2002, but our proof was of a completely different nature because this last formula has probably nothing to do with elliptic integrals. Hence, we tried and succeded with the Wilf-Zeilberger (WZ)-method, a powerful method of summation of hypergeometric series but of unknown reach (we cannot know in advance whether the method will work).

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