Colloquium 2017.

Celebrating Contributions of Antonio Campillo to Mathematics.

June 23, 2017, Valladolid, Spain

Abstracts

09:30–10:15

New insights on Hasse-Schmidt derivations.

Luis Narváez Macarro. Universidad de Sevilla.

Hasse-Schmidt derivations (HS-derivations, for short) are very natural objects associated with any commutative algebra. They may be of finite or infinite length, and those of length 1 coincide with classical derivations. In characteristic zero, the extension of classical derivations up to infinite length HS-derivations is always possible, and in some sense, this is the main reason because we can avoid the study of HS-derivations in such a case. However, in non-zero characteristic, HS-derivations contain many interesting phenomena and take part in some important results for which they seem to play a crucial role. In this talk, I will review on the basics of this theory and I will explain some new insights which are close to the "algebraic quantum point of view".

10:15–11:00

Desingularization of monoid schemes.

Guillermo Cortiñas. Universidad de Buenos Aires.

Although toric varieties over a field k as defined from a fan are always normal, the varieties one encounters in its desingularization process are not always normal. However one can desingularize in such a way so as to never leave the realm of monoid k-schemes, i.e. schemes locally of the form $\mathrm{Spec}(k[M])$ for some abelian monoid M. In order to treat desingularization in a characteristic-free manner, one introduces abstract (i.e. ground ring-free) monoid schemes. The basic idea is that the prime ideals of a (pointed) abelian monoid M form a space $\mathrm{Spec}(M)$ which is naturally equipped with a sheaf of monoids; these gadgets are the affine monoid schemes. General monoid schemes are obtained by gluing affine ones. For each commutative ring k there is a realization functor $X \mapsto X_k$ from monoid schemes to k-schemes, so that $(\mathrm{Spec}\, M)_k = \mathrm{Spec}(k[M])$. Most basic concepts in algebraic geometry have an analogue for monoid schemes, including blowup, smoothness, normally flat subschemes, etc. In the talk, based on joint work with Haesemeyer, Walker and Weibel ([2], [3]) we shall present a recent desingularization theorem for monoid schemes (which is itself a variant of a theorem of Bierstone and Milman [1]) via blowups along smooth, normally flat centers, and discuss some applications.

[1] Bierstone, Edward ; Milman, Pierre D. Desingularization of toric and binomial varieties. J. Algebraic Geom. 15 (2006), no. 3, 443–486.

[2] G. Cortiñas, C. Haesemeyer, M.E. Walker, C. Weibel. The K-theory of toric schemes over regular rings of mixed characteristic. arXiv:1703.07881

[3] G. Cortiñas, C. Haesemeyer, M.E. Walker, C. Weibel. Toric varieties, monoid schemes and *cdh* descent. J. reine angew. Math. 698, 1-54 (2015)

11:30–12:15

Flags in Vanishing Cohomology of the Milnor Fibre of a Hypersurface with an Isolated Singularity.

Xavier Gómez Mont. CIMAT, Guanajuato.

In Deligne's introduction of Mixed Hodge structures, a fundamental point is that the basic Mathematical Object of Linear Algebra is not a vector space, but it is a vector space provided with an increasing family of subvector spaces. It is what is called a flag of vector spaces. He showed then that the cohomology groups of a quasiprojective variety H^* contain 2 flags, F^* and W_* and that any algebraic morphism of quasiproyective

varieties $X \to Y$ induces a linear map $H^*(Y) \to H^*(X)$ on cohomology which preserve both flags, sending each subvectorsubspace to the corresponding subvectorsubspace and with further properties (the F-flags induced in $\mathrm{Gr}_W(H^*)$ is a pure Hodge structure).
This idea goes through in Singularity Theory, and the vanishing cohomology of an isolated hy- persurface singularity has a canonical mixed Hodge structure. This mixed Hodge structure brings to the forefront a finer invariant that the eigenvalues of the monodromy map, which is the spectrum, which brings in the FW-filtration. This filtration in cohomology may be dualized to give a filtration in vanishing homology. My interest lately has been to give a geometric interpretation of this filtration topologically in homology, where as the spectrum gives an analytic interpretaction of the filtration in cohomology.
Restricting to the simplest non-trivial case of plane curves, we can describe the spectrum using Puiseux pairs(the basic topological invariants of the curve singularity) and give a precise and delicate algorithm to write out the spectrum using "jumping numbers". This brings to the forefront another filtration in vanishing cohomology which is the polar filtration, which has as many terms as there are Puiseux pairs. The meaning of this filtration is that it is measuring the speed of vanishing of the vanishing cycles. This part of the lecture will report on research of Manuel Gonzalez-Villa, Carlos Guzman and myself.
In the case of a planar curve with spectrum of multiplicity 1, we will give a decomposition of the vanishing cohomology with real coefficients into an orthogonal (with respect to cup product) direct sum of 2 dimensional vector spaces and we will finish by posing the problems:

1. Obtain a geometric decomposition of the Milnor fibre reflecting this orthogonal direct sum decomposition of vanishing cohomology.
2. Interpret the geometric meaning of the flag of eigenspaces of the Monodromy corresponding to increasing angle of the eigenvalues.
3. Relate this increasing filtration to the form of vibrating strings with respect to increasing frequency of vibration.

12:15–13:00

Poincaré series: definitions, computation, connections.

Sabir Gusein-Zade. Moscow State University.

The Poincaré series of a multi-index filtration was first defined in a paper by A. Campillo, F. Delgado and K. Kiyek. Now this definition looks very natural, however, I cannot say that this was so from the very beginning. I'll discuss possible alternative definitions of the Poincaré series (applied when the initial one does not work), some methods to compute them and their relations with topological invariants (say, with monodromy zeta functions). A. Campillo participated in a big part of this history.

15:30–16:15

Ultrametric spaces of branches on normal surface singularities.

Patrick Popescu-Pampu. Université de Lille.

I will report about a joint work with Evelia García Barroso, Pedro González Pérez and Matteo Ruggiero. I will explain that on any complex normal surface singularity, there are arbitrarily large sets of branches which constitute naturally ultrametric spaces. Moreover, the whole set of branches is ultrametric if and only if the singularity is arborescent, that is, the dual graphs of its good resolutions are trees.

16:15–17:00

Curves and valuations. Generating sequences.

Félix Delgado de la Mata. Universidad de Valladolid.

For a finite set of plane valuations the semigroup of values, in general, is not finitely generated. However there exists a finite generating sequence. The use of HN expansions, a technical tool renowed by Campillo in his early work on singularities, together with results by Galindo and Campillo on the structure of the associated graduate rings provides the main ingredients to treat and understand the problem.