Abstrakt algebra med konkrete anvendelser 2
Aalborg Universitet (2013)
Spiseseddel 2

2. gang, fredag d. 5. september, 17:00–20:00 i lokale G5-112

- 17:00-19:00 Forelæsning: Gröbner bases (Kapitel 5 fra [Lau] og kapitel 21 fra [GG]).
- 19:00-20:00 Opgaveregning: A, B, C, D, E. Fra [GG]: 21.6, 21.2, 21.8, 21.7, 21.21, 21.23, 21.17, 21.9 (kun i). Fra [Lau]: 5.10, 5.13, 5.14, 5.15, 5.18, 5.19, 5.20, 5.21, 5.22.

  Exercise A: Let $R = \mathbb{F}_3[X, Y]$. Let $f = X^2Y + 2XY^2 + XY + X$, $f_1 = X + 2Y^2 + 1$, $f_2 = Y^2 + Y$. Divide f by $\{f_1, f_2\}$ considering the monomial order $<_{\text{lex}}$. Divide f by $\{f_1, f_2\}$ considering now the monomial order $<_{\text{grlex}}$.

  Exercise B: Read examples 21.1, 21.2, 21.3.

  Exercise C: Let $R = \mathbb{F}_3[X, Y]$. Let $f = X^2Y + 2XY^2 + XY + X$, $f_1 = X + 2Y^2 + 1$, $f_2 = Y^2 + Y$. Divide f by $\{f_1, f_2\}$ considering the monomial order $<_{\text{lex}}$. Divide f by $\{f_1, f_2\}$ considering now the monomial order $<_{\text{grlex}}$.

  Exercise D: Let $I = \langle f_1 = x^2y - 1, f_2 = xy^2 - x\rangle$ and consider the lex order.

  1. Show that $\{f_1, f_2\}$ is not a Gröbner basis for I.
  2. Trace the Buchberger algorithm for computing a Gröbner basis for I. You can use Sage or Maple for computing the S-polynomials.

  Exercise E: Consider the ideal in Example 21.21 and the Gröbner basis $\{f_1, \ldots, f_5\}$ computed in pages 598 and 599. Compute a minimal Gröbner basis this ideal using lemma 21.36. Use Maple or Sage to compute the reduced Gröbner basis of this ideal.

Med venlig hilsen,

Diego