Algebra 1 (2012)-Aalborg University
Lecture 24, November 27th

24th Lecture: Tuesday November 27th. I will not be present during this lecture.

8:15-12:00 Work in groups.

- Self-study. Simple groups and the 15-puzzle (pages 86–92). Examples 2.10.6, 2.10.9, 2.10.10 and Lemma 2.10.8.

- Exercises from previous lectures that you did not solved yet: Exercises from [Lau], 2.11 (page 104): 40 (hint: use $\tau = (1\ 2)$ and lemma 2.9.8 for (ii)), 51, A, B, C, 49, D, 41, 45, E.

  Exercise A (exam last year): Let $G = D_3 = \{e, a, a^2, b, ba, ba^2\}$, where $\mathrm{ord}(a) = 3$, $\mathrm{ord}(b) = 2$ and $aba = b$. Write G as a disjoint union of conjugacy classes. Compute $Z(G)$.

  Exercise B (exam last year): Prove that for $n \geq 3$, the center of S_n is $\{e\}$, i.e. $Z(S_n) = \{e\}$.

  Exercise C (exam last year):

  1. Let $\sigma \in S_{13}$,
  $$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\ 3 & 4 & 5 & 6 & 7 & 8 & 1 & 10 & 11 & 2 & 13 & 12 & 9 \end{pmatrix}.$$
     Compute the order and the sign of σ.
  2. Let $\sigma_1 = (1\ 4\ 2\ 5\ 3)$, $\sigma_2 = (1\ 3)(4\ 5) \in S_5$. Find $\tau \in S_5$ such that $\tau\sigma_1 = \sigma_2$.
  3. Let $\sigma' = (1\ 5)(2\ 4) \in S_5$. Compute the number of inversions of σ' and write σ' as a product of the minimal number of simple transpositions.

  Exercise D: Write all the elements of S_4, each factored into disjoint cycles. Compute A_4.

  Exercise E: Let $H = \{e, (1\ 2)(3\ 4), (1\ 3)(2\ 4), (1\ 4)(2\ 3)\}$. Show that H is a normal subgroup of A_4. Compute A_4/H and write down the composition table of A_4/H.

Best regards,

Diego