Some slides for 10th Lecture, Algebra 1

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A relation R on a set S is a subset $R \subset S \times S$. We say xRy to mean $(x, y) \in R$.

A relation R on S is

- reflexive if xRx for every $x \in S$
- symmetric if $xRy \implies yRx$ for every $x, y \in S$
- transitive if xRy and $yRz \implies xRz$ for every $x, y, z \in S$

R is called equivalence relation if it is reflexive, symmetric and transitive.

Example: $I \subset R$ an ideal in a ring. We define the relation:

$$x \equiv y (\mathrm{mod}\ I) \iff x - y \in I$$

- Reflexive: $0 \in I$
- Symmetric: $x \in I \implies -x \in I$
- Transitive: $x, y \in I \implies x + y \in I$.

Let $\sim$ be an equivalence relation on a set S. Given $x \in S$, set

$$[x] = \{s \in S : s \sim x\} \subset S$$

This subset is called the equivalence class containing x and x is called a representative for $[x]$.
The set of equivalence classes $\{[x] : x \in S\}$ is denoted $S/\sim$.

Example: In the previous example $R/\sim$ is equal R/I, where $\sim$ is $\equiv$.

Compare page 225 and page 63

- Lemma A.2.3 and Lemma 2.2.6 (ii)
- Corollary A.2.4 and Lemma 2.2.6 (iii)
- Theorem A.2.6 and Corollary 2.2.7
- Definition A.2.7 and Example 2.2.4 (page 68)
- Theorem A.2.8 and Theorem 2.5.1 (page 71)